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# SYNOPSIS ALGEBRAICA,

Opus Posthumum

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In Usum

Scholæ Mathematicæ

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OPERA & STUDIO suo
Emendatam, Auctam, & Revisam,
Officii ergô, Lubens, meritò
Editor *Inscribit.*

# Index Capitum.



LOGI-

Pag.



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# INTRODUCTIO
## AD
## Algebram Speciosam.

ALgebra Speciosa est Sapientia quantitatum, ad solvenda Problemata & investiganda Theoremata quæ in iis delitescunt. Speciosa dicitur ad differentiam Numerosæ, & ratio appellationis paulo post patebit. Cæterùm alio nomine vocatur Analytica, nec non Mathesis universalis.

Quantitates [sive sint Numeri sive Magnitudines] distinguuntur in Datas & Quæsitas.

Utræque denotari solent certis literis alphabeti, etsi apud diversos Authores diversimodè.

Nos quantitates datas designabimus per literas alphabeti priores, *a*, *b*, *c*, *d*, &c. quæsitas autem per postremas, *x*, *y*, *z*. Et quidem adhibendo literas minores, tam quia faciliùs pinguntur, tam verò ad vitandam confusionem; quoniam majores seu capitales literæ communiter adhiberi solent in designandis figurarum Geometricarum lineis.

Itaque quantitates iisdem literis designatæ [verbi gratiâ *a* & *a*; *b* & *b*] in eâdem quæstione intelliguntur ejusdem esse valoris & subjecti, hoc est,

denotant easdem seu æquales lineas vel numeros: ac proinde præfixis numeris in unum conjungi possunt, ut $2a$, $3b$, $4c$, $\frac{1}{2}a$, $\frac{2}{3}b$, $\frac{3}{4}c$, &c.

Quæ verò diversis literis designantur, sunt diversi valoris; quæ non conjunguntur, nisi per modum multiplicationis, aut mediantibus signis $+$ vel $-$; quorum prius denotat *plus*, alterum *minus*; illud est nota additionis seu augmenti, hoc vero subductionis seu defectus.

Ac notandum, quod signa $+$ & $-$ semper quantitatibus suis præponuntur, adeoque ad quantitatem sequentem referuntur; & quantitates quæ nullum signum præfixum habent [ut sunt initiales] intelliguntur habere $+$.

Inter quantitates diversi valoris potest tamen dari æqualitas, exempli gratiâ, ut duo $b$ sint æqualia tribus $a$ [sicut 5 floreni Belgici sunt æquales 9 solidis Anglicanis.]

Pro denotanda autem æqualitate [quæ in Algebra utramque paginam facit] adhiberi solent hæc signa; $=$ vel ꝏ; ut $2b=3a$; hoc est, $2b$ æqualia $3a$, &c.

Porro Instrumenta, quibus Algebra ad solvendas quæstiones & inquirendas veritates utitur, sunt tam operationes Logisticæ, tam præcipuè æquationes.

Logistica alia est Numerosa, quæ per Numeros; alia Speciosa quæ per Species seu Symbola universalia procedit, nempe per elementa alphabeti, quibus tum numeri tum magnitudines [uti initio dictum] designantur.

Hinc ipsa Algebra distinguitur in Numerosam & Speciosam: quarum illa vetustior, hæc recentior

tior est; & quantum constat, Franciscum Vietam primum Authorem agnoscit; à cujus tempore non tantum egregiè à diversis præstantissimis viris exculta, sed jam velut ad apicem perfectionis perducta est.

Discrimen autem inter utramque Algebram potissimum in eo consistit, quod Speciosa in quovis Problemate non duntaxat ipsum quæsitum invenit, sicut Numerosa; sed insimul generalem regulam & modum construendi in lucem profert; id quod Numerosa præstare non valet, dum hæc plerumque operationibus suis vestigia resolutionis confundit, quæ Speciosa semper integra conservat, ut eadem ad compositionem prompto pede repetere liceat.

Logistica Speciosa, perinde ut Numerosa, considerandа & tractanda venit in quantitatibus (1.) absolutis iisque integris, (2.) fractis seu fractionibus, (3.) surdis seu irrationalibus, (4.) binomiis.

Cæterum Logistica Numerosa hoc loco partim præsupponitur, partim ex Speciosa ultrò innotescet, cum utraque iisdem regulis utatur.

Sequitur ergo Logistica quantitatum integrarum.

# ADDITIO

Conjungit ſive copulat omnes quantitates propoſitas, ſervatis iiſdem Figuris $+$ & $-$.

## Exempla.

1. *In Quantitatibus ſimplicibus ſeu ſolitariis ejuſdem nominis.*

| | | | | | |
|---|---|---|---|---|---|
| | $2a$ | | $a$ | | $3d$ |
| add. | $a$ | add. | $-a$ | | $5d$ |
| ſumma | $2a+a$ | ſum. | $a-a$ | ſum. | $8d$ |
| hoc eſt | $3a$ | hoc eſt | $0$ * | | |

* Nam quantitates ejuſdem nominis contrariis ſignis adfectæ, in add. ſe mutuo tollunt.

| | | | | | |
|---|---|---|---|---|---|
| | $2c$ | | $5b$ | | $2b$ |
| add. | $4c$ | add. | $-2b$ | add. | $-5b$ |
| | $9c$ | ſum. | $5b-2b$ | ſum. | $2b-5b$ |
| ſum. | $15c$ | hoc eſt | $3b$ | hoc eſt | $-3b$ |

2. *In quantitatibus ſimplicibus diverſi nominis.*

| | | | | | |
|---|---|---|---|---|---|
| | $a$ | | $4c$ | | $3c$ |
| add. | $b$ | add. | $2d$ | add. | $2e$ |
| ſum. | $a+b$ | ſum. | $4c+2d$ | & | $f$ |
| | | | | ſum. | $3c+2e+f$ |

| | | | | | |
|---|---|---|---|---|---|
| | $b$ | | $3f$ | | $2d$ |
| add. | $-a$ | add. | $1$ | add. | $-2$ |
| ſum. | $b-a$ | ſum. | $3f+1$ | ſum. | $2d-2$ |

3. In

3. *In quantitatibus compoſitis ſeu copulatis, ejuſdem nominis.*

| | | | |
|---|---|---|---|
| | $2a + 3c + 4d$ | | $a + b - 2c$ |
| add. | $5d + a + 2c$ | add. | $a - b + 3c$ |
| ſum. | $3a + 5c + 9d$ | ſum. | $2a \quad + c$ |

Advertendum hic quantitates addendas interdum diſſimili ordine proponi, quæ tamen facile ſuo quoque loco reſtituentur. Idem quoque in Suſtractione & Diviſione uſu venire ſolet.

Ubi ſi ſigna ſint diverſa, additio fiet ſubſtrahendo, præpoſito ſigno majoris.

| | | | |
|---|---|---|---|
| | $6a + 2b - d$ | | $4f + 2b - 5c$ |
| add. | $-4d - 2a + 3b$ | add. | $3c - f - 2b$ |
| ſum. | $4a + 5b - 5d$ | ſum. | $3f \quad - \quad 2c$ |

| | | | |
|---|---|---|---|
| | $2aa+2b-5c+9d$ | | $bb+2abc-ab+ac$ |
| add. | $-9d+aa-6b+5c$ | add. | $bb-abc+ab-3ac$ |
| ſum. | $3aa \quad - \quad 4b$ | ſum. | $2bb + abc - 2ac$ |

| | | | |
|---|---|---|---|
| | $5d - 3ff + 2$ | | $3ff + 2d - 2$ |
| add. | $-2d + 4ff - 6$ | ad. | $-4ff - 5d + 6$ |
| ſum. | $3d + ff - 4$ | ſum. | $-ff - 3d + 4$ |

4. *In quantitatibus compoſitis diverſi nominis.*

| | | | |
|---|---|---|---|
| | $3a + c$ | | $5d - 4c + 1$ |
| add. | $2b + d - f$ | add. | $-a - 2b + 45$ |
| ſum. | $3a+c+2b+d-f$ | ſum. | $5d-4c-a-2b+46$ |

# SUBTRACTIO

COnjungit ſive copulat utraſque quantitates propoſitas, mutatis omnibus ſignis quantitatis ſubducendæ.

## Exempla.

1. *In quantitatibus ſimplicibus ejuſdem nominis.*

| | | | | | |
|---|---|---|---|---|---|
| ex | $5a$ | ex | $2bc$ | ex | $4acd$ |
| ſubt. | $2a$ | ſubt. | $bc$ | ſubt. | $4acd$ |
| reſtant | $5a - 2a$ | reſt. | $2bc - bc$ | reſt. | $4acd - 4acd$ |
| hoc eſt | $3a$ | hoc eſt | $bc$ | hoc eſt | $0$ |

| | | | | | |
|---|---|---|---|---|---|
| ex | $-df$ | ex | $4b$ | ex | $3c$ |
| ſubt. | $-df$ | ſubt. | $6b$ | ſubt. | $-3c$ |
| reſt. | $-df + df$ | reſt. | $4b - 6b$ | reſt. | $3c + 3c$ |
| hoc eſt | $0$ | hoc eſt | $-2b$ | hoc eſt | $6c$ |

2. *In quantitatibus ſimplicibus diverſi nominis.*

| | | | | | |
|---|---|---|---|---|---|
| ex | $a$ | ex | $3d$ | ex | $c$ |
| ſubt. | $b$ | ſubt. | $2c$ | ſubt. | $a$, $b$, & $d$ |
| reſt. | $a - b$ | reſt. | $3d - 2c$ | reſt. | $c - a - b - d$ |

| | | | | | |
|---|---|---|---|---|---|
| ex | $a$ & $b$ | ex | $df$ | ex | $4bc$ |
| ſubt. | $c$, $d$, & $e$ | ſubt. | $-2bd$ | ſubt. | $4$ |
| reſt. | $a+b-c-d-e$ | reſt. | $df + 2bd$ | reſt. | $4bc - 4$ |

*Nota.* Si dubium ſit, utra quantitatum ſit major vel minor, differentia generaliter alio quodam ſigno exhiberi ſolet, v.g.

$b \backsim c$ hoc eſt $b - c$ vel $c - b$

3. *In*

3. *In quantitatibus compositis ejusdem nominis.*

* EXEMPLA ubi signa sunt eadem.

| | | | |
|---|---|---|---|
| ex | $4a + 3b$ | ex | $6c - 5b$ |
| subt. | $2a + 2b$ | subt. | $4c - 3b$ |
| rest. | $2a + b$ | rest. | $2c - 2b$ |
| ex | $3c + 2a - b$ | ex | $5d + 12$ |
| subt. | $c + 3a - 2b$ | subt. | $6d - 5$ |
| rest. | $2c - a + b$ | rest. | $-d + 17$ |
| ex | $ab - 4cd$ | ex | $3c + 2a - d$ |
| subt. | $2ab - 5cd$ | subt. | $2c + 3a$ |
| rest. | $-ab + cd$ | rest. | $c - a - d$ |

| | | | |
|---|---|---|---|
| ex $4a - 2c$ | L sit $a = 9$ | ∵ $4a - 2c = 32$ |
| sub. $2a - 6c$ | $c = 2$ | & $2a - 6c = 6$ |
| rest. $2a + 4c$ | | & $2a + 4c = 26$ |
| ex $5d + 2b - c$ | L sit $d = 12$ | ∵ $5d + 2b = 66$ |
| sub. $4d + 5b - c$ | $b = 3$ | $4d + 5b = 63$ |
| rest. $d - 3b$ | | $d - 3b = 3$ |

| | |
|---|---|
| ex | $6a + 9b - 3c + 4d$ |
| subst. | $7a + 2b - 3c + 8d$ |
| restant | $-a + 7b \quad\quad - 4d$ |

* Ubi si signa sint eadem, erit & residuum idem ; nisi cum inferior quantitas est major, tunc à subtrahendo, vice versâ, residuum sortietur signum contrarium. Si vero signa sint diversa, subtractio fiet addendo, præfixo signo superioris.

EXEMPLA ubi ſigna ſunt diverſa.

| | | | |
|---|---|---|---|
| ex | $4a + 3b$ | L ſit $a = 3$ | $\because\ 4a + 3b = 18$ |
| ſub. | $2a - 2b$ | $b = 2$ | $2a - 2b = 2$ |
| reſt. | $2a + 5b$ | | $2a + 5b = 16$ |

| | | | |
|---|---|---|---|
| ex | $2b - 2c + d$ | ex | $5bc - bb$ |
| ſubt. | $b + 4c - d$ | ſub. | $3bb - 3bc$ |
| reſt. | $b - 6c + 2d$ | reſt. | $8bc - 4bb$ |

| | |
|---|---|
| ex | $aa - 4d + 6c - 2a$ |
| ſubt. | $4a + 2aa - 5c - 6d$ |
| reſt. | $2d - aa + 11c - 6a$ |

| | |
|---|---|
| ex | $6bb + 3aa - 3c + 9b$ |
| ſubt. | $4bb - 4aa - 6c - 5b$ |
| reſtant | $2bb + 7aa + 3c + 14b$ |

| | |
|---|---|
| ex | $ccc + bbd - aac - eee$ |
| ſubt. | $2bbd - 2ccc + eee$ |
| reſtant | $3ccc + bbd - aac - 2eee$ |

| | |
|---|---|
| ex | $7ab + dd - 10$ |
| ſubt. | $9ab - 2dd + 18$ |
| reſt. | $-2ab + 3dd - 28$ |

4. *In quantitatibus compositis diversi nominis.*

| | | | |
|---|---|---|---|
| ex | $aa - bb$ | ex | $bc - dd$ |
| ſubt. | $aa - ab + ac$ | ſubt. | $ad + bd - 2cd$ |
| reſt. | $ab - bb - ac$ | reſt. | $bc - dd - ad - bd + 2cd$ |

ex $5b - 2a$ ſubtr. $3c + d - 4f$
reſt. $5b - 2a - 3c - d + 4f$

| | |
|---|---|
| ſit $b = 10$ | $\because$ $5b - 2a = 40$ |
| $a = 5$ | $3c + d - 4f = 9$ |
| $c = 6$ | $5b - 2a - 3c - d + 4f = 31$ |
| $d = 7$ | |
| $f = 4$ | nam $5b + 4f = 66$ |
| | ſubt. $2a + 3c + d = 35$ |
| | reſt. $31$ |

MULTI-

# MULTIPLICATIO

Connectit [immediate] literas utriusque factoris; seu apponit multiplicantem ipsi multiplicando. At numeri quantitatibus præfixi multiplicantur more vulgari. Porrò signa eadem [sive + fuerit sive −] dant in producto +, sed signa diversa dant −.

E X E M P L A.

1. *In quantitatibus simplicibus.*

| | |
|---|---|
| Multipl. | $a$ |
| in | $b$ |
| product. erit | $ab$ |

| | | |
|---|---|---|
| Multipl. | $a$ | |
| in | $a$ | |
| product. | $aa$ vel $a^2$ | |
| hoc est | $a$ | quadratum |
| seu | $a$ | secundanum |

| | | |
|---|---|---|
| mult. | $aa$ seu $a^2$ | |
| in | $a$ | |
| prod. | $aaa$ seu $a^3$ | |
| hoc est | $a$ | cubicum |
| seu | $a$ | tertianum |

mult.

mult. $d$ in $b$ in $a$

$\overline{db}$

$a$

$\overline{dba}$ productum

mult. $5cd$ in $3ab$

prod. $15abcd$

mult. $2ac$ in $10$

prod. $20ac$

mult: $c$ in $12$

prod. $12c$

mult. $6b^4$ in $3b^2$

prod. $18b^6$

mult. $4a$

in $2b$

prod. $8ab$

mult. $4ab^2$ in $4ca^2$

prod. $16a^3b^2c$

mult. $9aacdf$

in $5accf$

prod. $45a^3c^3f^2d$

mult. $3bd$

in $2ab$

$\overline{6bbad}$

mult: $c^3$ in $5d^2$

prod. $5d^2c^3$

Ubi accedit productorum partialium additio seu collectio [more vulgari] ut habeatur productum totale.

mult. $a+b$ in $c$

prod. $ac+bc$

mult. $b-a$ in $2d$

prod. $2bd-2ad$

mult. $a+2b-3c+d$ in $3e$

prod. $3ae+6be-9ce+3de$

mult. $6b-2a-c-4f$ in $2ab$

prod. $12ab^2-4a^2b-2abc-8abf$

mult. $c+b$ in $a+d$

prod. $ac+ab+cd+bd$

mult.

mult. $b - a$ in $c - d$
prod. $bc - ac - bd + ad$
mult. $a + b$ in $a + b$
prod. $a^2 + 2ab + b^2$
mult. $a - b$ in $a - b$
prod. $a^2 - 2ab + b^2$
mult. $b + c$ in $b - c$
prod. $b^2 - c^2$
mult. $b + c - 2d$ in $b + e - 2d$
prod. $b^2 + 2bc - 4bd + c^2 - 4cd + 4d^2$
mult. $2a + 4b$ in $3c - 5d$
prod. $6ac + 12bc - 10ad - 20bd$
mult. $6f - d - 2$ in $f + 2d$
prod. $6f^2 + 11df - 2d^2 - 2f - 4d$
mult. $3b^2 - 2a^2 + 5$ in $9a + 4b - 1$
prod. $27ab^2 - 18a^3 + 45a + 12b^3$, &c.

Cæterum notandum multiplicationem interdum ſymbolice tantum indigitari hoc modo.

$$a + b - c \times d + e$$

vel $\overline{a + b - c}$ in. $\overline{d + e}$

vel ita $a + b - c : d + e$

DIVI-

# DIVISIO

Resolvit dividendum in divisorem & quotum: quo in totum divisorem ducto restituatur exhauriaturque ipse dividendus. Cæterùm cum numeris præfixis divisio instituitur more vulgari, & signa eadem dant in quoto +, diversa dant −, ut in multiplicando.

Vel Divisio subducit quasi seu extrahit divisorem ex dividendo, & quod relinquitur assumit pro quoto.

Et hoc quidem sufficit cum divisor est simplex; sed si is fuerit compositus, inquisitio quoti tantum instituitur cum prima nota divisoris, perinde ut in praxi numericâ fieri solet.

### Exempla.

1. *In quantitatibus simplicibus.*

divid. $aa$ per $a$ | $a$ quotus. divid. $bc$ per $b$ | $c$ quotus.

divid. $2abc$ per $2b$ | $ac$ quotus.

divid. $9cd$ per $3c$ | quotus $3d$

divid. $3f$ per $f$ | quotus $3$

divid. $4a^2b^2$ per $2b^2$ | quotus $2a^2$

divid. $6c^3b^4$ per $2c^2b^3$ | quotus $3bc$

divid. $3a^2b^3$ per $3ab^2$ | quotus $ab$

divid. $18cd$ per $9$ | quotus $2cd$

divid. $4a^3b^2c$ per $5a^2b$ | quotus $\frac{4}{5}abc$ seu $\frac{4abc}{5}$

divid. $6b^2c^3d^4$ per $4bc^2d^3$ | quotus $1\frac{1}{2}bcd$

divid. $12a^4c^4d^3$ per $4a^2c$ | quotus $3a^2c^3d^3$

divid. $5df$ per $5df$ | quotus erit 1. Nam si divisor sit=dividendo, quotus semper erit unitas.

2. *In*

2. *In quantitatibus compositis.*

divid. $ab + bc$ per $b$ | quotus erit $a + c$
divid. $bc - cd$ per $c$ | quotus $b - d$
divid. $af + ag + df + dg$ per $f + g$ | quot. $a + d$
divid. $bd - bc - ad + ac$ per $d - c$ | quotus $b - a$
div. $6a^2b + 12abc - 10a^2c^4 - 20ac^5$ per $2a^2 + 4ac$ | qu. $3b - 5c^4$
divid. $d^2 + d$ per $d + 1$ | quotus $d$
divid. $2f^3 - f^2$ per $2f - 1$ | quotus $f^2$
divid. $a^2 - b^2$ per $a + b$ | quotus $a - b$
divid. $a^2 - b^2$ per $a - b$ | quotus $a + b$
divid. $b^3 - 3b^2a + 3ba^2 - a^3$ | quot. $b^2 - 2ab + a^2$
per $b - a$

ſubtr. $b^3 - b^2a$

reman. $-2b^2a + 3ba^2 - a^3$ | $-2ba$
$b - a$

ſubtr. $-2b^2a + 2ba^2$

reman. $+ba^2 - a^3$ | $+a^2$
$b - a$

ſubtr. $+ba^2 - a^3$

div. $8ab^3 - 12b^2cd + 16b^6 + 10abc^3 - 15c^4d + 20b^4c^3$
per $4b^2 + 5c^3$

1. membrum] $8ab^3 + 10abc^3$ | $2ab$
2. membrum] $-12b^2cd - 15c^4d$ | $-3cd$ quotus
3. membrum] $+16b^6 + 20b^4c^3$ | $+4b^4$

[Examen per multiplicationem.]

divid. $c^3 + d^3$ per $c + d$ | quotus $c^2 + d^2 - cd$
divid. $ab + 3b - 2a - 6$ per $b - 2$ | quotus $a + 3$

Porro

Porro si divisio modo præcedenti fieri nequeat, subscripto divisore ipsi dividendo constituetur fractio, ut patet ex sequentibus,

$$\text{divid.}\left\{\begin{array}{l} ac \text{ per } b \\ ef \text{ per } bc \\ c \text{ per } d \\ def \text{ per } gb \end{array}\right\}\text{quotus erit}\left\{\begin{array}{l} \frac{ac}{b} \\ \frac{ef}{bc} \\ \frac{c}{d} \\ \frac{def}{gb} \end{array}\right.$$

Sic $a+b$ per $d$, erit $\frac{a+b}{d}$

$b^2+c^2$ per $e+f$ $\quad \frac{b^2+c^2}{e+f}$

$\frac{2a+3c-d}{b-c-d}$ per $b+c+d$

$dbc+af$ per $bc$ | $d+\frac{af}{bc}$

$bde+3c$ per $d$ | $be+\frac{3c}{d}$

$efg+fg$ per $ef$ | $g+\frac{fg}{ef}$ hoc est $\frac{g}{e}$

§ Pari modo divisio quævis symbolice indigitari solet, etiamsi illa reipsa fieri possit, ut

$$\frac{a^2-b^2}{a+b}$$

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REGULA

# Regula PROPORTIONUM

EX datis duobus vel tribus terminis ritè diſpoſitis, ope multiplicationis & diviſionis, perinde ut in praxi numeroſa, eruit tertium vel quartum proportionalem quæſitum.

Et quantitatum in ſerie proportionali diſpoſitio denotari ſolet hoc modo;

| | (1) | | (2) | | (2) | | (3) |
|---|---|---|---|---|---|---|---|
| | *a*. | | *b* :: | | *b*. | | X |
| hoc eſt, ut | *a* | ad | *b* : | ita | *b* | ad | X |

| | (1) | | (2) | | (3) | | (4) |
|---|---|---|---|---|---|---|---|
| | *a*. | | *b* :: | | *c*. | | X |
| ut | *a* | ad | *b* : | ita | *c* | ad | X |

Vel etiam ſic;

*a* ÷ *b* : *b* ÷ X

*a* ÷ *b* : *c* ÷ X

Porro quoad ipſam praxin notandum

1.) Si in termino primo & ſecundo; vel in primo & tertio, eædem literæ occurrant, illas per modum diviſionis ante operationem expungi; quod ipſum etiam de præfixis numeris intelligendum.

2.) Si vero primus terminus ſit diverſus à reliquis [ſaltem ex parte] uti plerumque fit, quartum quæſitum ſemper in forma fractionis prodire

EXEMPLA

### *Exempla in Quantitatibus simplicibus.*

Termini dati.

1.) $a.$ $\underbrace{2a :: \quad 2a}$

$\frac{4\cancel{a}a}{\cancel{a}}$ | $4a$ tertius quæsitus.
quo præsuppos. $= x$
erit $x = 4a$

*Vel compendiosius sic:*

$\cancel{a}.$ $\underbrace{2\cancel{a} :: \quad 2a}$

$4\cancel{a}a$ tertius quæsitus.

2.) $b.$ $\underbrace{ab :: \quad ab}$

$\frac{aabb}{b}$ | $aab$ tertius quæsitus.

vel ita, $\cancel{b}.$ $\underbrace{a\cancel{b} :: \quad ab}$

| $aab$

3.) $2c.$ $\underbrace{d :: \quad d}$ | $\frac{dd}{2c}$ tertius quæsit.

4.) $\cancel{3}a.$ $\underbrace{b :: \quad \cancel{3}b}$ | $\frac{bb}{a}$ tertius quæsit.

5.) $\cancel{a}.$ $\underbrace{c :: \quad 2\cancel{a}b}$

$2bc$ quartus quæsitus.

6.) $2\cancel{b}.$ $\underbrace{3a :: \quad \frac{\cancel{4}\cancel{b}d}{\cancel{2}}}$

$6ad$ quartus quæsitus.

7.) $d.$ $\underbrace{5b :: \quad a}$ | $\frac{5ab}{d}$ quart. quæsit.

8.) $x\varnothing a^2 \quad 7c :: \quad x\varnothing d^2 \;\Big|\; \frac{7cd^2}{a}$ quart. quæſit.

*Exempla in Quantitatibus compoſitis.*

9.) $a. \quad a+b :: \quad a+b$

div. $\frac{aa+2ab+bb}{a} \;\Big|\; a+2b+\frac{bb}{a}$ tertius quæſitus.

10.) $a: \quad a+b :: \quad a-b$

div. $\frac{aa-bb}{a} \;\Big|\; a-\frac{bb}{a}$ quartus quæſitus.

11.) $b. \quad b+c :: \quad 2b-c$

div. $\frac{2bb+bc-cc}{b} \;\Big|\; 2b+c-\frac{cc}{b}$ quartus quæſitus.

12.) $a+2b \quad a :: \quad 2a-b$

$\Big|\; \frac{2aa-ab}{a+2b}$ quart. quæſit.

13.) $3b-3a \quad 2a+c :: \quad 3d$

$\frac{2ad+cd}{b-a}$ quartus quæſitus.

14.) $2d+c \quad 2b+d :: \quad 2d-c$

$\frac{4bd+2dd-2bc-cd}{2d+c}$ quart. quæſit.

GENE-

# GENESIS & ANALYSIS POTESTATUM.

[*Ab aliis Involutio & Evolutio dictæ, adhibitis hisce notis* @ *w* ]

POteſtates nihil aliud ſunt quam producta ex continuâ alicujus aſſumpti lateris ſeu radicis multiplicatione orta.

Nam ſuppoſito, quod quælibet radix ſit ſui ipſius prima poteſtas; vel, quod prima poteſtas oriatur, cum radix ducitur in unitatem;

Si deinde eadem radix ducatur

| | |
|---|---|
| in | ſeipſam |
| | ſuum Quadratum |
| | ſuum Cubum |
| | ſuum Quadrato-quadratum |
| | ſuum Quadrato cubum |

| | | | |
|---|---|---|---|
| produ-citur e-juſdem | Quadratum | quæ eſt pote-ſtas | Secundana |
| | Cubus | | Tertiana |
| | Quadrato-quadr. | | Quartana |
| | Quadrato-cubus | | Quintana |
| | Cubo-cubus | | Sextana |

| | | | | |
|---|---|---|---|---|
| quarum proinde notæ ſeu indices numerales ſunt | 2 | vel quælibet aliæ ex his ortæ | literales vero hæ | *Q* |
| | 3 | | | *C* |
| | 4 | | | *Qq* |
| | 5 | | | *Qc* |
| | 6 | | | *Cc* |

Et ſic porro in infinitum, pro numero ſcilicet dimenſionum ſuarum, ex quibus componuntur.

Quo autem magis compositæ sunt potestates, eò censentur altiores; ac proinde non tantum denominatione sed simul gradibus suis invicem distinguuntur: qui ipsi denique scalam aliquam progressionalem constituunt, in qua quantitates velut ascendant ac descendant.

Sed & notandum, quod Potestates quæ sunt supra Cubicam, non tantum ab ipsâ radice, in gradum proxime præcedentem ductâ, sed etiam ex potestatum inferiorum mutuâ multiplicatione procreentur, quod & ipsa nomina indicant. Exempli gratiâ, Quadrato quadratum oritur non tantum ex ducta radicis in cubum, sed etiam ex multiplicatione quadrati in seipsum; quò fit ut hæc potestas [quartana] etiam sit quadratum. Idem esto judicium de cæteris.

Porro omnis radix vel est (1) singularis, hoc est unius nominis, ut $a$, vel $2a$, vel $3a$, &c. nam numeri literis præfixi non augent nomen, vel (2) composita ex pluribus nominibus hoc est binomia, trinomia, &c. ut $a+b$, vel $a-b$, $a+b+c$, $a+b-c$, $a-c+b$, $a-b-c$, &c.

Deinde omnis radix vel est rationalis, ut $a$, $2a$, $\frac{1}{2}a$, &c. vel irrationalis, ut $\sqrt{a}$, $\sqrt{ab}$, & quales sunt radices, tales etiam sunt earum potestates; hoc autem loco tantum de rationalibus agitur.

*Genesis* alicujus potestatis, est ejusdem ex datâ radice per multiplicationem procreatio.

*Analysis* alicujus potestatis est radicis ex datâ potestate [ope divisionis peculiaris] extractio.

Itaque in *Genesis* datur radix, & quæritur potestas. In *Analysi* contra datur potestas & quæritur radix.

Tum

Tum vero radix alicujus potestatis [utut rationalis] non semper est latus proprie dictum seu linea [vel potestas prima] sed sæpe ipsæ potestates [ortæ] loco & nomine radicis veniunt. Sic *aa*, seu $a^2$ [potestas secundana] potest inservire pro radice cujuscunque potestatis constituendæ, &c.

Itaque Genesis potestatum à radice singulari facilis est: Nimirum (1) si radix sit latus [seu potestas prima] apponatur eidem [superius versus dextram] nota numerica simplex, optatæ potestati competens, tanquam multiplicationis requisitæ index: Ut si radix sit $a$, ejus quadratum erit $a^2$; cubus $a^3$, &c. Si radix sit $2a$, quadratum erit $4a^2$, cubus $8a^3$, &c. Numeri enim literis præfixi multiplicantur seorsim, ut suo loco dictum.

Si vero (2) radix sit potestas [orta] tum index ejusdem multiplicetur per indicem [simplicem] potestatis constituendæ, hoc est, pro constituendo quadrato duplicetur, pro cubo triplicetur, &c. ut si radix sit $a^2$, ejus quadratum erit $a^4$, cubus $a^6$, quadrato quadratum $a^8$, &c.

Intelligitur hinc, quod indices potestatum alii sint simplices seu primi, ut 2, 3, 4, &c. alii ex prioribus per multiplicationem compositi, primis suis æquivalentes; sic index quadrati seu potestatis secundanæ non tantum est 2, sed quilibet numerus ex binario compositus, ut 4, 6, 8, &c. Quo fit ut eadem quantitas diversos potestatum gradus subire possit; sic $a^6$ non tantum est potestas sextana, sed etiam tertiana, & secundana, &c.

Quod si denique (3.) ex duabus vel pluribus potestatibus ab eadem radice ortis, alia potestas componenda sit, fiet hoc, indices datarum potestatum addendo, & aggregatum ipsi radici superimponendo. Dentur exempl. gr. potestates $a^2$ & $a^3$; potestas ex his conflata erit $a^5$. Sic ex $a^3$, $a^4$, & $a^5$ prodibit $a^{12}$.

Neque difficilis est, etsi operosior, potestatum procreatio à radice composita, puta binomia, &c.

$$
\begin{array}{lll}
\text{Sit ex. gr.} & \text{radix} & a+b \\
 & & \underline{a+b} \ \text{multipl.} \\
 & & a^2+ab \\
 & & \underline{\quad +ab+b^2} \\
 & \text{erit} & a^2+2ab+b^2 \ \text{Quadratum} \\
 & & \underline{a+b} \ \text{mult.} \\
 & & a^3+2a^2b+ab^2 \\
 & & \underline{\quad +a^2b+2ab^2+b^3} \\
 & & a^3+3a^2b+3ab^2+b^3 \ \text{Cubus} \\
 & & \underline{a+b} \\
 & & a^4+3a^3b+3a^2b^2+ab^3 \\
 & & \underline{\quad +a^3b+3a^2b^2+3ab^3+b^4} \\
 & & a^4+4a^3b+6a^2b^2+4ab^3+b^4 \ \text{Quadrato-quadratum.}
\end{array}
$$

Atque hoc modo à radicibus compositis diversæ potestatum tabulæ construi possunt, quousque libuerit.

Cæterum

Cæterum potestates à radice quâcunque compositâ productæ per singulos gradus habent suas certas proprietates, tum quoad ipsas partes, ex quibus componuntur, earumque genituras, tum quoad signa + & —. De quibus & aliis huc spectantibus videatur Clavis *Oughtredi.*

Sic fuit potestatum *Genesis*; quâ probe intellectâ in praxi speciosâ [adhibitis, in quantum fieri potest, figuris] proclivis etiam via erit ad Analysin, quæ nihil aliud est quam Inventio seu Extractio radicis ex quâlibet potestate propositâ; quin eadem ad praxin numerosam loco regulæ & declarationis, magno sane verborum compendio, inserviet.

---

# EXTRACTIO
# Radicis Quadratæ.

[*Cujus nota est* $\sqrt{q}$. *vel etiam* $\sqrt{}$ *absolute.*]

### (1.) *Ex quantitatibus simplicibus.*

Quæ fit dividendo indicem potestatis propositæ per 2, & si quadratum numericum præfixum sit, ex eo seorsim radicem extrahendo.

$$\text{Ex}\begin{cases} a^2 \\ b^2 \\ c^2 \end{cases}\text{rad. quadr. est}\begin{cases} a \\ b \\ c \end{cases} \qquad \text{Ex}\begin{cases} 4\,a^2 \\ 9\,b^2 \\ 16\,d^2 \end{cases}\sqrt{q}.\text{ est}\quad \begin{matrix} 2\,a \\ 3\,b \\ 4\,d \end{matrix}$$

$$\text{Sic ex}\begin{cases} a^2b^2 \\ c^4 \\ 25d^6. \end{cases}\text{rad. quadr. est}\quad \begin{matrix} ab \\ c^2 \\ 5d^3 \end{matrix}$$

### (2.) *Ex Quantitatibus Compositis.*

Ubi Praxis ita se habet:

$$\begin{array}{ll} \text{Extr. } \sqrt{q}.\text{ ex } a^2+2ac+c^2 & \\ \qquad\qquad a^2+2a\ \ +c & \Big|\ a+c \text{ rad. quæsita.} \\ \qquad\qquad\quad \underline{c}\qquad \underline{\text{mult.}} & \\ \qquad\qquad 2ac+c^2 \text{ subtr.} & \end{array}$$

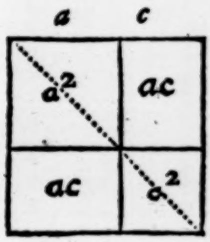
1. Po-

1.) Ponatur radix Quadrati diagonalis $a^2$, quæ eſt $a$, & ipſum quadratum $a^2$ deleatur per modum Subtractionis.

2.) Duplicetur inventa radicis nota $a$, duplum erit $2a$, quod ſcribatur ſub $2ac$ [aggregato complementorum] fiat diviſio [$2ac$ per $2a$] quotus $c$, erit altera nota tum radicis, tum etiam diviſoris, hactenus imperfecti, ut nimirum totus diviſor ſit $2a+c$: qui demum multiplicetur in quotum ſuum $c$, & productum ſubtrahatur ex ſupraſcripto; (3.) ubi cum tandem nihil relinquitur, indicium eſt Quadrati rationalis & radicis rite inventæ.

[*Nota:* Duplicatio radicis vocari ſolet genitura, puta diviſoris requiſiti; ſicut in extr. √ cub. triplicatio quadrati, &c.]

3.) Quod ſi porro plures adhuc notæ radicis eruendæ reſtent [ut in Exemplo ultimo] duplicatio notarum præcedentium inventarum dictaque ſimul diviſio continuetur ad finem uſque; ubi cum tandem nihil relinquitur.




Ratio operationis facile patet ex ipſa Geneſi & appoſitâ Figurâ. Nam primam notam radicis dat Quadratum diagonale $a^2$; quo ablato reſtat gnomon, conſtans ex duplici complemento, $ac$ & altero diagonali $c^2$. Cum igitur utrumque complementum contineatur ſub utrâque parte radicis [hoc eſt ſub $a$ & $c$, ſequitur

tur ſi aggregatum complementorum [ $2ac$ ] dividatur per primam notam radicis duplicatam ut [ $2a$ ] quod tunc in Quoto emerget altera radicis nota [ $c$ ]. Quæ etſi in praxi ſpecioſâ ex ipſo ſtatim altero diagonali innoteſcat, tamen in praxi numeroſâ ope dicti diviſoris [ $2a$ ] inquirenda eſt & acquirenda ; utut diviſor hic ſit imperfectus uſque dum ipſe Quotus eidem jungatur.

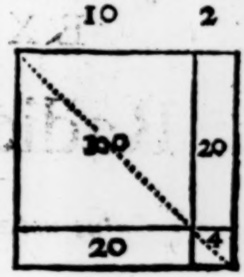
§. Ex $a^2 - 2ab + b^2$ | $a - b$ √ quæſita.
$a^2 + 2a - b$
$-b$ mult.
$-2ab + b^2$ ſubtr.

§. Ex $4a^2 + 8ab + 4b^2$ | $2a + 2b$ √ quæſ.
$4c^2 - 16cd + 16d^2$ | $2c - 4d$ √ quæſ.

§. Ex $b^2 + 2bc + c^2 + 2bd + 2cd + d^2$ | $b + c + d$ √ quæſita.
$b^2 + 2b + c$ mult.
$c$
$+2bc + c^2$ ſubtr.
$+2b + 2c + d$ mult.
$+ d$
$+2bd + 2cd + d^2$ ſubtr.

*Applicatio*

## *Applicatio dictorum in Numeris.*




Sit extrahenda $\sqrt{q}$. ex 144 quæ quidem nota eſt nimirum 12, hæc ita diviſa intelligatur, ut prima nota (1.) pro valore loci ſui [10] conſtituat unam partem, altera vero [2.] reliquam, & tunc procedatur ut ſupra.

| | | |
|---|---|---|
| 144 | \| | 10+2 hoc eſt 12 $\sqrt{}$ quæſita. |
| 100 | | 10 |
| 44 | | 2 |
| $2\cancel{0}$ | | 12 |
| 2 | | |
| 44 | | |

Cæterum idem obtinebitur [etſi non pari commoditate] radice 12 diviſa utcunque, puta in 9 & 3, in 8 & 4, &c.

EXTRA-

# EXTRACTIO
# Radicis Cubicæ.

[*Cujus nota est hæc,* $\sqrt{}$ cub. *vel* $\sqrt{c}$.]

(1.) *Ex Quantitatibus simplicibus.*

Quæ fit dividendo indicem potestatis propositæ per 3, & si Cubus numericus præfixus sit, ex eo seorsim radicem cubicam extrahendo.

$$\text{Ex}\left\{\begin{array}{c} a^3 \\ b^3 \\ 8c^3 \\ 27d^3 \end{array}\right. \text{rad. cub. est}\left\{\begin{array}{l} a \\ b \\ 2c \\ 3d \end{array}\right.$$

$$\text{Ex}\left\{\begin{array}{c} a^3d^3 \\ 125b^3c^3 \\ 64b^6 \end{array}\right. \text{rad. cub. est}\left\{\begin{array}{l} ad \\ 5bc \\ 4b^2 \end{array}\right.$$

(2.) *Ex Quantitatibus compositis.*

Ubi Praxis ita se habet:

$$\begin{array}{lll} \text{Extr. } \sqrt{c}.\text{ ex } b^3+3b^2c+3bc^2+c^3 & | & b+c\ \sqrt{c}.\text{ quæs.} \\ \quad b^3+3b^2\quad +3b & | & \\ \text{mult. } (+c \quad\ +c^2 & | & +c^3 \\ \hline \quad +3b^2c+3bc^2+c^3 & & \end{array}$$

1.) Ponatur

1.) Ponatur radix Cubica Cubi diagonalis $b^3$, quæ eſt $b$, & ipſe Cubus $b^3$ per ſubtractionem deleatur.

2.) Inventa radicis nota $b$ triplicetur, & triplum $3b$ ſcribatur ſub $3bc^2$, tanquam termino ſeu complemento correſpondente. Tum idem triplum $3b$ multiplicetur per dictam radicis notam $b$, productum erit $3b^2$, quod ſcribatur ſub complemento correſpondente $3b^2$; ſic habebitur pars diviſoris. Dividatur itaque $3b^2c$ per $3b^2$, quotus $c$ erit altera nota radicis: Quâ multiplicatâ tum in triplum præcedentis notæ $b$, ut fiat $3bc$, tum in ſeipſam ut fiat Quadratum $c^2$, emerget totus diviſor $3b^2+3bc+c^2$, qui demum multiplicetur in quotum ſuum $c$, & productum ſubtrahatur ex ſupraſcripto. [Vel quod idem erit, inventa radicis nota $c$, ducatur in $3b^2$, & ejus Quadratus $c^2$ in $3b$, quibus denique jungatur ejuſdem Cubus $c^3$.]

3.) Si plures notæ radicis eruendæ reſtent, triplicentur notæ inventæ omnes, & triplum ſubſcribatur locis convenientibus, tum idem triplum multiplicetur per omnes notas radicis & productum convenienter ſubſcribatur, nimirum pro monitu ipſarum literarum; fiatque diviſio & cætera, ut ſupra *Num.* 2. iterando hanc ipſam operationem quoties opus, uſque ad finem: ubi cum tandem nihil relinquitur, indicium eſt Cubi rationalis & radicis cub. recte inventæ, cujus inſuper examen habebitur per geneſin, inventum ſcil. radicem rurſum cubice multiplicando.

Ratio

Ratio operationis patet ex Genesi & adhibendâ Figurâ solidâ. Nam primam notam radicis exhibet cubus diagonalis $b^3$, quo ablato restat gnomon, constans ternis complementis $b^2c$ [similibus & æqualibus] itemque ternis $bc^2$; tum altero cubo diagonali $c^3$. Cum igitur prior complementorum ordo [$b^2c$] contineatur sub Quadrato prioris notæ radicalis [$b$] & sub ipsa altera nota $c$; sequitur si horum complementorum aggregatum [$3b^2c$] dividatur per $3b^2$ [hoc est] per triplum radicis $b$ ductum in radice $b$, seu quod idem est, per radicis Quadratum triplicatum, quod tunc in quoto emerget altera radicis nota $c$, &c.

§ Ex $a^3-3a^2b+3ab^2-b^3$ | $a-b$ $\sqrt{c}$ quæsit.

$$
\begin{array}{ll}
 & a^3+3a^2+3a \\
\text{mult.} & -b \quad +b^2 \\
\hline
 & -3a^2b+3ab^2-b^3
\end{array}
$$

§ Ex $8c^3-12c^2d+6cd^2-d^3$ | $2c-d$ $\sqrt{c}$ quæs.

$$
\begin{array}{ll}
 & 8c^3+12c^2+6c \\
\text{mult.} & (-d \quad +d^2 \\
\hline
 & -12c^2d+6cd^2-d^3
\end{array}
$$

*Applicatio in Numeris.*

Detur numerus 12167, cujus radix cubica 23, hæc ita divisa intelligatur ut prima nota (2) pro valore loci sui [10] constituat unam partem [20] altera vero [3] reliquam; hoc est radix 23 sit divisa in 20+3, quo supposito procedatur ut sequitur.

subtr.

$$12167\left\{\begin{matrix} b+c \\ 20+3 \end{matrix}\right.$$

ſubtr. $8\cancel{0}\cancel{0}\cancel{0}$

| | | | mult. |
|---|---|---|---|
| | | 4167 | |
| | $3b$ | $\cancel{60}$ | $20=b$ $60=3b$ $3=c$ \| $180=3bc$ |
| add. | $3b^2$ | 1200 | $400=b^2$ $1200=3b^2$ \| $9=c^2$ |
| | $3bc$ | 180 | |
| | $c^2$ | 9 | |
| | | 1389 | totus Diviſor. |
| | | mult. 3 | Quotus. |
| ſubtr. | | 4167 | |

EXTRA-

# EXTRACTIO
# Rad. Quadrato-quad.

[*Cujus nota est hæc,* $\surd qq$, *vel* $\surd\surd$ ]

### (1.) *Ex Quantitatibus simplicibus.*

Quæ fit dividendo indicem potestatis propositæ per 4, & si Q. quadratum numericum præfixum sit, ex eo seorsim $\surd\surd$ extrahendo.

Ex $\left\{\begin{matrix} a^4 \\ b^4 \end{matrix}\right.$ $\surd qq.$ est $\left\{\begin{matrix} a \\ b \end{matrix}\right.$ Ex $\left\{\begin{matrix} b^4 \\ d^4 \end{matrix}\right.$ $\surd\surd$ est

### (2.) *Ex Quantitatibus compositis.*

Ubi praxis ita se habet:

1.) Ex $b^4 + 4b^3c + 6b^2c^2 + 4c^3b + c^4 \mid b + c$ $\surd\surd$ quæs.

$$\begin{array}{llllll} & b^4 + & b^3 + & b^2 + & b & \text{mult.} \\ & & 4 & 6 & 4 & \\ \hline \text{mult.} & & +4b^3 & +6b^2 & +4b & \\ & & +c & +c^2 & +c^3 & \\ \hline & & +4b^3c & +6b^2c^2 & +4c^3b & \mid \; +c^4 \end{array}$$

1.) Ponatur $\surd\surd$ Quadrato-quadrati diagonalis $b^4$, quæ est $b$; & ipsum $b^4$ per subtractionem deleatur.

2.) Inventa

2.) Inventa radicis nota $b$; ejus Quadratum $b^2$; necnon Cubus $b^3$ ſubſcribantur ordine à dextra verſus ſiniſtram locis correſpondentibus, & multiplicetur, $b$ quidem per 4, $b^2$ per 6, & $b^3$ rurſum per 4, ſemper; productum erit $4b^3+6b^2+4b$, conſtituens diviſorem pro ſequenti radicis nota, ex parte. Dividatur itaque $4b^3c$ per $4b^3$; Quotus $c$ erit altera nota radicis: qua multiplicata in $6b^2$, ut fiat $6b^2c$; & ejus Quadrato $c^2$ in $4b$, ut fiat $4bc^2$; tum etiam in ſeipſam cubice ut fiat $c^3$, orietur totus diviſor $4b^3+6b^2c+4bc^2$: Qui demum totus multiplicetur in ſuum Quotum $c$, & productum ſubtrahatur ex ſupraſcripto. (Vel quod idem erit) acquiſitis terminis $4b^3$, $+6b^2+4b$; multiplicetur $4b^3$ per $c$, $6b^2$ per $c^2$, & $4b$ per $c^3$, hoc eſt, terminus cubicus per ipſam notam radicalem $c$; terminus quadraticus per radicis Quadratum $c^2$; & terminus radicalis per radicis Cubum $c^3$; quibus denique jungatur ejuſdem Quadrato-quadratum $c^4$; eritque totum Collectum = ſuperiori.

3.) Si plures notæ Radicis, eruendæ reſtent, repetatur eadem operatio, *Num.* 2. aſſumendo omnes radicis notas prius inventas, &c. Ubi cum tandem nihil relinquitur, indicium eſt Quadrato-quadrati rationalis, & radicis recte inventæ.

Ratio operationis perinde ut in præcedentibus manifeſta fiet; niſi quod figuræ geometricæ ultra poteſtatem cubicam ſe non extendunt.

D 2.) Ex

2.) Ex
$$\begin{array}{l|l}
a^4 - 4a^3b + 6a^2b^2 - 4b^3a + b^4 & a - b \ \surd qq. \\
a^3 + 4a^3 \ + 6a^2 \ + 4a & \\
- \ b \ + b^2 \ - b^3 \ \text{mult.} & \\
\hline
-4a^3b + 6a^2b^2 - 4b^3a & + b^4
\end{array}$$

*Praxis in Numeris.*

Pari quoque methodo procedet, facilius tamen expedietur per geminam radicis Quadratæ extractionem, hoc eſt, Si ex numero propoſito primum extrahatur radix quadrata, & ex hac inventa radice iterum radix quadrata ; hæc ipſa enim [ſecunda $\surd q.$] erit $\surd qq.$ ipſius numeri biquadratici propoſiti.

Detur, exempli gratia, numerus 38416, hujus radix quadrata erit 196 : Et hujus radix quadrata 14. Itaque 14 erit radix Quadrato-quadratica numeri propoſiti 38416.

Fuit hactenus (quantum hic loci ſufficere poteſt) de extractione Radicum ex poteſtatibus rationalibus, hoc eſt ubi requiſita praxis locum habet, & nihil relinquit.

Cæterum cum extractio prædicto modo fieri non poteſt, indicium eſt, quantitatem vel poteſtatem propoſitam eatenus eſſe irrationalem : cui proinde duntaxat ſignum radicis expetitæ præfigitur ; & cum quantitas eſt compoſita ſignum iſtud per omnes terminos continuatur, & nomen Radicis Univerſalis accipit. Atque hæc eſt origo quantitatum Irrationalium ſive Surdarum, de quibus infra ſuo loco. Itaque, exempli gratiâ,

Radix

$$\text{Radix Quadr.}\left\{\begin{array}{l} a^3 \\ ab \\ 5b^2c \\ 3c^4 \end{array}\right\}\text{erit}\ \begin{array}{l} \sqrt{a^3} \\ \sqrt{ab} \\ \sqrt{5b^2c} \\ \sqrt{3c^4} \end{array}$$

[Quoniam index [3] non potest exactè dividi per 2.

[Hoc loco quidem index [4] potest dividi per 2, sed numerus præfixus 3 non est Quadratus.

---

$$\text{Radix Cubica}\left\{\begin{array}{l} a^4 \\ 8a^2b \\ 6c^3 \\ 5d^2 \end{array}\right\}\text{erit}\ \begin{array}{l} \sqrt{c.}\ a^4 \\ \sqrt{c.}\ 8a^2b \\ \sqrt{c.}\ 6c^3 \\ \sqrt{c.}\ 5d^2 \end{array}$$

---

$$\begin{array}{c}\text{Item} \\ \text{Radix Quadr.} \\ \text{universalis}\end{array}\left\{\begin{array}{l} a^2+b^2 \\ a^2+c^2-b^2 \\ 4b^3+bc^2 \end{array}\right\}\text{erit}\ \begin{array}{l} \sqrt{a^2+b^2}\ * \\ \sqrt{a^2+c^2-b^2} \\ \sqrt{4b^3+3bc^2} \end{array}$$

* Hoc est addantur duo Quadrata, $a^2$ & $b^2$, & ex aggregato extrahatur radix quadrata.

---

$$\begin{array}{c}\text{Rad. Cubica} \\ \text{universalis}\end{array}\left\{\begin{array}{l} a^3+8b^3 \\ 2a^3-b^3 \\ 3b^2c+2bc^2-c^3 \end{array}\right\}\text{erit}\ \begin{array}{l} \sqrt{c.\ a^3+8b^3} \\ \sqrt{c.\ 2a^3-b^3} \\ \sqrt{c.b^2c+2bc^2-c^3} \end{array}$$

# II. LOGISTICA FRACTIONUM.

## IN QUA,

*Uberioris exercitii gratiâ, conjungemus Praxin Numerosam cum Speciosa.*

### CAP. I.

### Notatio Fractionum.

§. *Fractiones propriæ:*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Numerator, | 1 | 2 | 3 | $a$ | $2a$ | $a^2$ | Supposito $b$ majori quam $a$. |
| Denominator, | 2 | 3 | 5 | $b$ | $3b$ | $b^2$ | |

§. *Fractiones impropriæ.*

$$\frac{2}{2}\ \Big|\ \frac{3}{2}\ \Big|\ \frac{9}{4}\ \Big|\ \frac{12}{5}\ \Big\|\ \frac{a}{a}\ \Big|\ \frac{b}{a}\ \Big|\ \frac{3b}{2a}\ \Big|\ \frac{2b^2}{a}$$

Vid. & *Cap.* II. *Memb.* I.

Ubi *Not.* Cum Denominator & Numerator ſunt $=$, Fractio ſemper eſt $=1$, &c.

§. *Fractiones*

§. *Fractiones mixtæ cum integris.*

$3\frac{1}{2}$ | $5\frac{3}{4}$ | $8\frac{5}{8}$ || $a+\frac{a}{b}$ | $2b-\frac{c^2}{3a}$

§. *Fractiones fractionum, ſeu Fractiones ſecundæ.*

$\frac{1}{2}$ ex $\frac{3}{4}$ hoc eſt, dimidium ex tribus quartis. Quod etiam ſcribi ſolet, $\frac{1}{2}, \frac{3}{4}$, interjecto nimirum commate, item $\frac{2}{3}$ ex $\frac{4}{5}$, duæ tertiæ ex quatuor quintis.

In his, poſterior fractio ſeu terminus, eſt principalis, & major, qui refertur ad ipſum totum: Sed prior terminus, qui eſt minor, refertur ad poſteriorem.

Sic in literis: $\frac{a}{b}$ ex $\frac{2a}{3b}$ Sitque $a=3$. $b=4$.

---

## CAP. II.

## Reductio ſeu Tranſmutatio Fractionum.

### §. 1. MEMBRUM.

*Fractiones improprias ad integra reducere.*

Dividatur Numerator per Denominatorem, Quotus dabit integra, &c. Reducantur.

ex. gr. 1.) $\frac{12}{4}$ | 3 integra quotus. *Sint Fractiones propoſitæ:*

2.) $\frac{8}{3}$ | $2\frac{2}{3}$ quotus. *Erunt reductæ.*

3.) $\frac{56}{7}$ | 8 quotus.

4.) $\frac{65}{12}$ | $5\frac{5}{12}$ quotus.

Sic in literis: $\frac{2aa}{a}$ | $2a$ quotus | $\frac{6bc}{2c}$ | $3b$ quot. | $\frac{a}{a}$ | 1

## §. 2. Membrum.

*Integra in formam Fractionis redigere.*

1.) Si integra ſint ſola, nec detur certus denominator, ſubſcribatur propoſito numero vel quantitati unitas, & factum erit.

| *Dentur quantitat. Integr.* | *Orientur Fractiones.* |
|---|---|
| $6$ | $\frac{6}{1}$ |
| $15$ | $\frac{15}{1}$ |
| $1$ | $\frac{1}{1}$ |
| $a$ | $\frac{a}{1}$ |
| $2b$ | $\frac{2b}{1}$ |
| $3ab$ | $\frac{3ab}{1}$ |

2.) Si integra ſint ſola, & detur certus denominator Fractionis optatæ.

Multiplicetur quantitas propoſita per datum denominatorem, & hic deinde ſubſcribatur producta.

Sit numerus 7 redigendus in fractionem, cujus denominator, ſit 4, fient $\frac{28}{4}$.

[Quod

[Quod quidem nihil aliud eſt ac ſi dicam, ſeptem ulnas vel libras integras, reſolvendas eſſe in Quadrantes.]

Detur quantitas $b$, & denominator fractionis $a-b$, orietur fractio impropria $\frac{ab-bb}{a-b}$

Detur quantitas $2a+c$, & denominator $f$, orietur fractio $\frac{2af+cf}{f}$

3.) Si integra habeant annexam fractionem, multiplicetur quantitas integra per denominatorem fractionis adhærentis; producto addatur numerator, & ſubſcribatur aggregato denominator datus.

| | | | | |
|---|---|---|---|---|
| Dentur, | $2\frac{3}{4}$ | $3\frac{2}{5}$ | $9\frac{4}{10}$ | $12\frac{6}{25}$ |
| | $\begin{matrix}8\\3\end{matrix}$ | $\begin{matrix}15\\2\end{matrix}$ | $\begin{matrix}90\\4\end{matrix}$ | $\begin{matrix}\underline{12}\\50\\25\end{matrix}$ |
| Fiunt, | $\frac{11}{4}$ | $\frac{17}{5}$ | $\frac{94}{10}$ | $\frac{306}{25}$ |

Detur $c+\frac{de}{a}$ prodibit $\frac{ac+de}{a}$

Detur $b+c+\frac{b^2+c^2}{b-c}$ prodibit $\frac{2b^2}{b-c}$

Detur $2ab-\frac{a^2c}{b}$ prodibit $\frac{2abb-a^2c}{b}$

## §. 3. Membrum.

*Fractiones in terminis majoribus propoſitas, ad minores terminos æquivalentes (ſi fieri poſſit) reducere.*

Poteſt enim quælibet fractio, una eademque innumeris modis exprimi, inter quos notiſſimus ille eſt qui terminos habet minimos.

Interim non omnis fractio ad minores terminos reducibilis eſt, ſed illæ duntaxat, in quibus numerator & denominator communem aliquam diviſorem, vel etiam plures, agnoſcant.

Si igitur quæratur utriuſque termini (hoc eſt numeratoris & denominatoris) communis menſura maxima (ut infra docebitur) & per hanc uterque dividatur, quoti dabunt terminos correſpondentes minimos.

Vel quod idem erit, fiant utrobique diviſiones continuæ, per eoſdem diviſores quoſcunque, qui ſe offerunt, ac nihil relinquunt, uſque dum nullus amplius communis diviſor occurrat niſi unitas.

Itaque in ſpecie (potiſſimum quoad Praxin Numericam) hoc loco obſerventur caſus ſequentes.

1.) Cum denominator poteſt dividi exacte per numeratorem, tunc ipſe numerator eſt communis diviſor maximus. Et hoc in caſu, numerator fractionis reductæ ſemper eſt unitas.

Dentur Fractiones

$$\frac{6}{24} \quad \frac{5}{15} \quad \frac{8}{40} \quad \frac{4}{32} \quad \frac{7}{63}$$

Prodibunt his æquivalentes in terminis minimis.

$$\frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{8} \quad \frac{1}{9}$$

Quod ſi autem illud ipſum non ſtatim appareat, cum ſcil. numeri ſunt majores, tunc procedendum erit juxta ſequentia.

2.) Si

2.) Si terminus uterque sit numerus par, bipartire eos quousque licet.

Detur fractio $\frac{32}{64}$ per continuam dimidiationem orientur hæ, $\frac{16}{32}$ | $\frac{8}{16}$ | $\frac{4}{8}$ | $\frac{2}{4}$ | $\frac{1}{2}$ quarum singulæ æquivalent fractioni datæ; sed ex his $\frac{1}{2}$ tanquam terminis minimis constans, est notissima.

Item detur fractio $\frac{16}{24}$; per dimidiationem prodibunt huic æquivalentes, $\frac{8}{12}$ $\frac{4}{6}$ $\frac{2}{3}$

3.) Si uterque terminus habeat ad dextram unam vel plures nullitates, deleantur utrobique numero pares [quod nihil aliud est quam per 10, vel 100, &c. utrinque dividere.]

Dentur $\frac{30}{40}$ vel $\frac{300}{400}$ vel $\frac{3000}{4000}$

abjectis utrinque totidem nullitatibus fiunt $\frac{3}{4}$

Dentur $\frac{400}{7000}$ prodeunt primum $\frac{4}{70}$ &
porro dimidiando $\frac{2}{35}$

4.) Si termini aliter se habeant, nec tamen numeri sint valde magni, facile etiam apparebit [ex tabula Pythagorica] si aliquem vel aliquos communes divisores admittant, cum quibus proinde operatio instituatur ut dictum.

Detur fractio $\frac{42}{63}$; hic primùm uterque terminus potest dividi per 7, unde prodeunt $\frac{6}{9}$, & hic porro uterque potest dividi per 3, ut prodeant $\frac{2}{3}$.

Item

Item detur fractio $\frac{15}{35}$; dividendo utrobique per 5, prodibunt $\frac{3}{7}$.

Item detur fractio $\frac{48}{54}$; dividendo per 6 habebitur æquivalens $\frac{8}{9}$.

5.) Si dictis modis res non ſuccedat, puta cum numeri ſunt majores, tunc demum communis menſura maxima inquiratur hoc modo.

Dividatur ſemper terminus major per minorem, tam in reſiduis quam ipſis datis, hoc eſt, ſemper diviſor præcedens per ſuum reſiduum, nulla habitâ ratione quotorum: Ultimus diviſor, qui nihil relinquit, erit utriuſque communis menſura maxima. [*Vid. Exemplum* A.]

Quodſi autem in hujuſmodi diviſione tandem remaneat unitas, indicium eſt, terminos fractionis propoſitæ nullum communem diviſorem agnoſcere præter unitatem, adeoque fractionem iſtam ad minores terminos reduci non poſſe. [*Vid. Exemplum* B.]

*A*] Detur fractio $\frac{105}{432}$ | Div. (12 reſid.

$$\begin{array}{r|l} 432 & \\ 1)\ 105 & 4 \end{array}$$

$$\begin{array}{r|l} (9 & \\ 2)\ 105 & 8 \\ 12 & \end{array} \qquad \begin{array}{r|l} (3 & \\ 3)\ 12 & 1 \\ 9 & \end{array} \qquad \begin{array}{r|l} 4)\ 9 & 3 \\ 3 & \end{array}$$

remanet nihil.

& ultimus diviſor erat 3, hic igitur eſt communis diviſor maximus pro reducenda fractione data, ut ſequitur.

| *Numerator.* | | *Denominator.* | | *Fractio reducta.* |
|---|---|---|---|---|
| Div. $\cancel{1}\cancel{0}\cancel{5}$ | 35 | $\cancel{4}\cancel{3}\cancel{2}$ | 144 | $\frac{35}{144}$ |
| $\cancel{3}$ | | $\cancel{3}$ | | |

*B*] Proponatur autem porro hæc ipſa fractio acquiſita $\frac{35}{144}$, puta ad minores terminos reducendo.

| | | | | | |
|---|---|---|---|---|---|
| | | (3 | | (1 rem.un. | |
| Div. 1.) $\cancel{1}\cancel{4}$(4 | 4 | 2.) $\cancel{3}\cancel{5}$ | 8 | 3.) $\cancel{4}$ | 1 |
| $\cancel{3}\cancel{5}$ | | $\cancel{4}$ | | $\cancel{3}$ | |

unde colligitur, fractionem hanc $\frac{35}{144}$ jam in minimis ſuis terminis ſtare, nec ulteriorem reductionem admittere.

§. In Praxi Specioſa, ſi quantitates non ſint nimis compoſitæ, communis diviſor * facile agnoſcitur, [dummodo quis in praxi Multiplicationis & Diviſionis ſatis ſit exercitatus] quo habito, eodem modo proceditur ut ſupra in numeris, dividendo ſcilicet utrumque terminum per communem diviſorem maximum repertum, & aſſumendo quotos loco propoſitorum.

* *Si quem habet.*

Cum autem quantitates valde ſunt compoſitæ ac prolixæ, inquiſitio communis diviſoris operoſior eſt & tædioſior, quam ut hoc loco [inter elementa] tradi & tractari poſſit.

Sufficient igitur quæ ſequuntur exempla.

*Fractiones*

| Fractiones propositæ. | Communes Divisores. | Fractiones reductæ. |
|---|---|---|
| $\frac{abc}{bd}$ | $b$ | $\frac{ac}{d}$ |
| $\frac{4a^2b^2}{2abc}$ | $2ab$ | $\frac{2ab}{c}$ |
| $\frac{abd^2}{b^2d}$ | $bd$ | $\frac{ad}{b}$ |
| $\frac{2a^3b^4c}{6ab^2c}$ | $2ab^2c$ | $\frac{a^2b^2}{3}$ |
| $\frac{b^2c+b^2d}{cd+d^2}$ | $c+d$ | $\frac{b^2}{d}$ |
| $\frac{a^2+2ab+b^2}{a^2-b^2}$ | $a+b$ | $\frac{a+b}{a-b}$ |
| $\frac{f^2c-f^2d-e^2c+e^2d}{dc-d^2}$ | $c-d$ | $\frac{f^2-e^2}{d}$ |
| $\frac{c^2-d^2}{bc+bd+c^2+cd}$ | $c+d$ | $\frac{c-d}{b+c}$ |
| $\frac{a^3+b^3}{a^2-b^2}$ | $a+b$ | $\frac{a^2+b^2-ab}{a-b}$ |

DE

# Æquationum Algebraicarum

## *Inventione, Distinctione, Reductione, Resolutione.*

QUotiescunque Problema aliquod sive quæstio proponitur, putetur præstitum esse quod postulatur: Et pro quantitate quæsita (sive sit numerus sive magnitudo) ponatur *x*, *y*, vel *z*, hoc est, una ex ultimis literis Alphabeti; pro datis autem priores, ut *a*, *b*, *c*, &c. quo facilius data à quæsitis dignoscantur.

Deinde Quantitates tam datæ quam quæsitæ secundum conditionem quæstioni convenientem tractentur atque comparentur, addendo, subtrahendo, multiplicando, dividendo, donec tandem aliquid inveniatur, quantitati de qua quæritur, vel suæ ad quam adscendet, potestati æquale.

Quoniam autem in omni fere æquatione, ubi primùm ex involucris quæstionis emergit, nota cum ignotis miscentur ac confunduntur: Ideo termini cujusque æquationis ita sunt ordinandi ac reducendi ut data faciant unam partem, & quæsita alteram. Tum verò Quantitates utrinque ad formam lateris seu radicis redigendæ sunt. Id quod reductionem æquationum ad simplicissimam formam vocamus; quæ his potissimum axiomatis innititur.

Si

Si æqualibus æqualia addantur vel ſubducantur, aggregata ac reſidua erunt æqualia.

Si æqualia per æqualia multiplicentur vel dividantur, quæ oriuntur erunt æqualia.

Æqualium radices homogeneæ, erunt æquales.

Porro æquationes non ſunt unius generis, ſed tot earum exiſtunt claſſes, quot gradus poteſtatum ſeu dimenſionum in ſcala Quantitatum progreſſionali. Unde oriuntur æquationes,

$$\left.\begin{array}{l}1.\ \textit{Simplices,}\\ 2.\ \textit{Quadraticæ,}\end{array}\right\}\left\{\begin{array}{l}3.\ \textit{Cubicæ,}\\ 4.\ \textit{Quadrato-quadr.}\ \ \&c.\end{array}\right.$$

Quarum ſingulæ ſingulas quaſi Algebræ partes conſtituunt.

Reductio Æquationum (potiſſimum ſimplicium ſeu primarum) ad ſimpliciſſimam formam perficitur,

1. *Additione.*

$$\begin{array}{lrcl}\text{Sit} & x-a & = & b+c\\ \text{add.} & +a & & +a\\ \hline \text{erit} & x & = & b+c+a\end{array}\qquad\qquad \begin{array}{rcl}c-x & = & 0\\ +x & & +x\\ \hline c & = & x\end{array}$$

$$\begin{array}{rcl}Q.\ -b & = & 0\\ +b & & +b\\ \hline Q. & = & b\end{array}$$

2. *Subtractione*

2. *Subtractione.*

$$\begin{array}{lll} \text{Sit } Q. + a = b + c & & x + a - b = 2x + d \\ \text{subt. } \underline{+a} & \underline{+a} & \underline{x} \text{ subtr.} \\ \text{erit } Q. = b + c - a & & a - b = x + d \\ & & \text{subt. } \underline{+d} \quad \underline{+d} \\ & & a - b - d = x \end{array}$$

[*Nota:* Additionis itaque & Subtractionis hujus compendium erit Transpositio Quantitatum ex una parte æquationis in alteram facienda semper sub signo contrario.]

$$\begin{array}{l} \text{Sit } a - b - x = b + x \\ \text{erit } a - 2b = 2x \end{array}$$

3. *Multiplicatione.*

$$\begin{array}{ll} \text{Sit } \left(\begin{array}{l} \frac{x}{a} = b \\ \underline{a} \text{ mult. } \underline{a} \end{array}\right) & \frac{Q}{3b} = a - c \\ \text{erit } \frac{\cancel{a}x}{\cancel{a}} = ab & \text{mult. per } 3b \\ \text{hoc est } x = ab & \text{erit } Q = 3ab - 3bc \end{array}$$

4. *Divisione.*

$$\begin{array}{ll} \text{Sit } x^2 = 6x & 3x^2 = 9x \\ \underline{x \text{ div.}} & \underline{3x \text{ div.}} \\ \text{erit } x = 6 & x = 3 \end{array}$$

Sit

$$\begin{array}{lll} \text{Sit } x^2+ax & = & bx+cx \\ \quad x \text{ div.} & & \\ \hline \text{erit } x+a & = & b+c \\ \& \quad x & = & b+c-a \end{array}$$

5. *Radicum extractione.*

$$\begin{array}{ll} \text{Sit } x^2 = a^2 & \text{Sit } y^2 = ab \\ \text{extr. } \surd & \text{extr. } \surd \\ x = a & y = \sqrt{ab} \end{array}$$

$$\begin{array}{l} \text{Sit } x^2 + ax + \tfrac{1}{4}a^2 = b^2 \\ \text{extr. } \surd \\ x + \tfrac{1}{2}a = b \\ \& \ x = -\tfrac{1}{2}a + b \end{array}$$

$$\begin{array}{l} \text{Sit } x^3 - 3bx^2 + 3b^2x - b^3 = c^3 \\ \text{extr. } \surd c \\ x - b = c \\ \& \ x = b + c \end{array}$$

SE-

## SEQUUNTUR
# Problemata *seu* Quæstiones:
### *Sub genere Æquationum simplicium.*

1. INvenire numerum, quo multiplicato per 3, ex producto subtractis 5, residuoque diviso per 2, si quoto huic addatur ipse numerus quæsitus, aggregatum sit 40.

Ponatur numerus quæsitus $=x$; eritque

1. *Praxis Numerosa.*

Num. quæsitus $=x$
mult. per $\underline{3}$
product. erit $3x$
hinc subtr. $5$
resid. erit $3x-5$
quod div. per $2$
quotus erit $\frac{3x-5}{2}$
huic add. num. quæsitus $x$
aggregatum erit $\frac{3x-5}{2}+x = 40$
quare multiplicando erit $3x-5+2x = 80$
hoc est $5x-5 = 80$
& transponendo $5x = 80+5$
hoc est $5x = 85$
& dividendo per $\underline{5}$
$x = 17 =$ numero quæsito.

 §. *Examen*,

§. *Examen.*

| | |
|---|---|
| Numerus inventus | 17 |
| multipl. per | 3 |
| | 51 |
| subtr. | 5 |
| Divid. per | 46 |
| | 2 |
| | 23 |
| add. | 17 |
| provenit | 40 quod erat propositum. |

2. *Praxis Speciosa.*

Sit 3 seu numerus multiplicans $= a$
5 subtrahendus $= b$
2 divisor $= c$
40 aggregatus $= d$
Et hi omnes erunt dati vel assumti pro lubitu.

| | | |
|---|---|---|
| Jam igitur: | Num. quæsitus | $= x$ |
| | mult. per | $a$ |
| | product. | $ax$ |
| | subtr. | $b$ |
| | resid. | $ax - b$ |
| | divid. per | $c$ |
| | quotus | $\frac{ax-b}{c}$ |
| | add. | $x$ |
| | aggregat. | $\frac{ax-b}{c} + x = d.$ |

ergo multiplicando: $ax - b + cx = cd$
& transponendo: $ax + cx = cd + b$
& dividendo per $a + c$

$$x = \frac{cd+b}{a+c}$$

Hinc

Hinc regula pro hujuſcemodi Quæſtionibus generalis: Numerus æquationis datus [ut h. 1. 40] multiplicetur per datum diviſorem: Producto addatur numerus ſubtrahendus: Totum hoc dividatur per multiplicantem auctum diviſore; Quotus dabit numerum quæſitum.

§. Ubi obſervandum, quod inventio quæſiti perficiatur operatione ſeu methodo ipſi quæſtioni contraria. Et ſic quoque in cæteris.

2. Invenire numerum, quo multiplicato per 12, & ad productum additis 48, tantundem proveniat, ac ſi idem numerus quæſitus per 18 eſſet multiplicatus.

*Vel generaliter ita,*

Invenire numerum, quo multiplicato per $a$, & ad productum addito $b$, tantundem proveniat, ac ſi idem numerus eſſet multiplicatus per $c$.

3. Invenire numerum, cui ſi addantur 11, & ab eodem [numero primo] ſi ſubſtrahantur 7, ſumma additionis ſit dupla reſidui ſubſtractionis.

*Vel generaliter.*

Invenire numerum, cui ſi addatur $b$, & ab eodem [numero primo] ſi ſubſtrahatur $c$, ſumma additionis ſit ad reſiduum ſubtractionis in ratione data, puta dupla, tripla, quadrupla, &c.

4. Reperire numerum, cui ſi addatur ipſius duplum [triplum, quadruplum, &c.] proveniat ejuſdem numeri quadratum.

5. Invenire numerum qui si ad seipsum addatur: summa per eundem multiplicetur: ex producto idem numerus subtrahatur: & tandem residuum hoc per eundem dividatur, ut proveniant 13.

6. Dividere numerum 16 in duas partes, ita ut quadratum majoris partis excedat quadratum minoris per 32.

7. Dividere numerum 36 in duas partes, ita ut si ad primam addantur 12, & ad alteram 6, summa prior sit dupla posterioris.

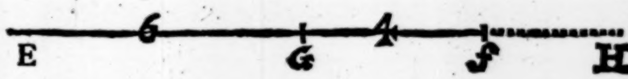
8. Sit linea AB (partium 70) secta utcunque in C [ita ut AC sit 42, BC 28] oportet eandem lineam AB aliter secare, v. gr. in D, ita ut rectangulum ADC sit æquale quadrato DB quæritur intersegmentum CD [quo habito innotescent AD, DB].


*9. Vel ita in Numeris.*

Sint duo numeri 42 & 28: inveniendus est tertius, quo addito ad 42, si totum hoc multiplicetur per ipsum numerum quæsitum, tantundem proveniat, ac si eodem numero quæsito subtracto ex 28, residuum multiplicetur quadratè.

9. Sit

9. Sit linea EF, ſecta utcunque in G [ita ut EG ſit 6, GF 4] oporteat hanc rectam EF producere [v. gr. in H] ita ut rectangulum EHF ſit æquale quadrato GH; quæritur prolongatio FH.




### §. *Vel ita in Numeris.*

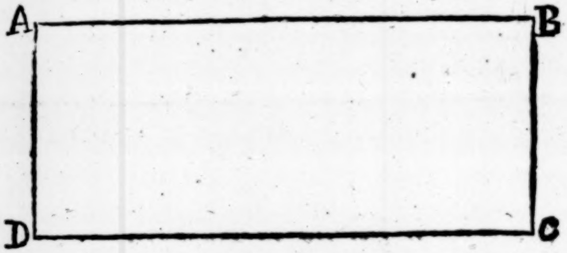
Sint duo numeri 6 & 4: inveniendus eſt tertius, quo addito ad ſummam utriuſque datorum, ſi totum hoc multiplicetur per quæſitum, tantundem proveniat, ac ſi aggregatum ex quæſito & minore datorum multiplicetur quadratè.

10. Dux bellicus dum exercitum ſuum in aciem quadratam diſponit, ſuperſunt ipſi milites 284; at cum ſingulas lineas uno milite auget [ſeu cum latus quadrati unitate majus conſtituit] deſunt ipſi ad explendum quadratum 25 milites: quæritur quot milites habuerit.

11. Capitaneus quidam emittit $\frac{1}{3}$ ſuorum militum $+10$, reſtant ipſi $\frac{1}{2}+15$. Quær. quot mil. habuerit.

12. Eſt exercitus, cui ſi addatur ſui $\frac{1}{2}$, $\frac{2}{3}$, & $\frac{3}{4}$ $-5000$, ſumma erit 100000. Quær. numerus iſtius exerc.

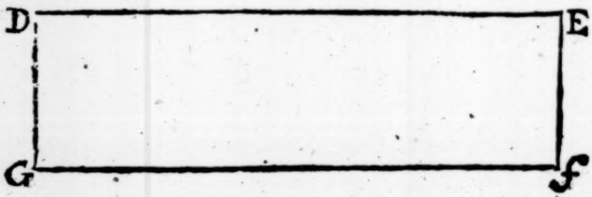
13. In rectangulo ABCD, differentia lateris majoris AB, & lateris minoris BC, est 12, differentia autem quadratorum à dictis lateribus, est 1680. Quæruntur latera rectanguli ABCD.




*Hoc est, proponendo generaliter.*

Datâ in rectangulo differentiâ laterum, & differentiâ quadratorum laterum, invenire latera rectanguli.

14. Rectanguli DEFG longitudo DE est dupla latitudinis EF; & summa quadratorum longitudinis ac latitudinis est decupla summæ ipsorum laterum DE, EF. Quæruntur latera rectang. DEFG.




*Seu generaliter.*

Data in rectangulo ratione laterum, & ratione summæ quadratorum laterum, ad summam ipsorum laterum, invenire latera rectanguli.

15. Inve-

15. Invenire duos numeros in ratione, ut 2 ad 3 quorum productum, si invicem multiplicentur; sit 54.

Sit minor numerus $= X$

$\because 2 \;\cdot\!\!-\!\!\cdot\; 3 : X \mid \frac{3X}{2} =$ majori numero.

vel, sit communis factor [quo in terminos datæ rationis ducto proveniant ipsi numeri desiderati] $= X$ : $\because$ numerorum quæsitorum minor $= 2X$ & major $= 3X$

16. Invenire duos numeros, quorum ratio ad invicem sit ut 4 ad 5 : & summa quadratorum à singulis, sit 2624.

17. Invenire latus quadrati, cujus area sit ad summam laterum in data ratione, puta ut 45 ad 12.

18. Invenire latus cubi, cujus superficies ad soliditatem sit in data ratione, puta ut 6 ad 11.

19. Quidam conducit operarium, ea conditione ut singulis diebus quibus operi institerit, accipiat 12 Stufros, sed quibus otiatus fuerit, mulctetur 8 Stufris. Elapsis autem diebus 390 neuter alteri quicquam debet. Quæritur quot diebus fuerit operatus, quot otiatus. 3 *s.* 1 *s.*

20. Dominus quidam conducit Servum, cui pro mercede annuâ pollicetur 24 Florenos, unà cum pallio. Finitis autem 8 mensi- 24 *l.*

bus Servus impetrato discessu, mercedis loco accepit pallium + 13 Flor. Quæritur quanti constiterit pallium.

13 *l.*

21. Quidam interrogatus quot annos haberet, respondit, Si annorum meorum $\frac{2}{3}$ quadruplicem, & producto addam $\frac{1}{2}$ eorundem + 50; summa in tantum supra 100 excrescet, quantum nunc infra 100 subsistit annorum meorum numerus.

22. Rogatus quidam quota diei hora esset, respondit, Dies hoc tempore est horarum 16; si jam $\frac{1}{2}$ horarum elapsarum addantur $\frac{2}{3}$ residuarum, prodibit hora quæsita ab ortu Solis numeranda.

23. A Norimberga ad Romam sunt 140 milliaria: eodem tempore ex singulis his urbibus exit viator, quorum unus quolibet die confecit milliaria 8, alter 6. Quæritur quot ab initio itineris diebus elapsis sibi obviam venient, & quot milliaria singuli perfecerint.

24. Tabellarius aliquis conficit singulis diebus milliaria 6: post 8 dies insequitur hunc alius, qui singulis diebus conficit 10 milliaria. Quæritur quoto die priorem assequetur.

25. Quidam Tabellarius conficit singulis diebus milliaria 6: postquam vero 56 milliaria perfecit, insequitur eum alter, qui singulis diebus 8 conficit. Quæritur quot diebus hic illum assequetur.

26. Qui-

26. Quidam coëmit tres libros, in ratione pretii, ut 12, 5, 1: Si pretium primi duplicetur, secundi triplicetur, tertii quadruplicetur; summa horum productorum in tantum superabit 10 imperiales, quantum summa pretium maximi & medii est infra 5 imperiales. Quær. quanti constiterint dicti libri.

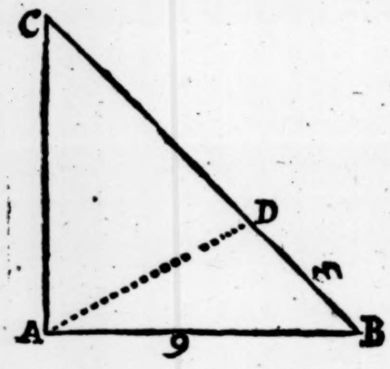
27. Sit numerus 50 secandus in duas partes, ita ut parte majore divisâ per 7, & minore multiplicatâ per 3, summa hujus producti & prioris quoti restituat eundem numerum propositum 50.

28. Sit numerus 20 secandus in duas partes, ita ut Quadratum minoris partis subductum ex Quadrato majoris relinquat ipsum numerum propositum 20 [vel, relinquat duplum, triplum, &c. numeri propositi.

29. Si quis singulis septimanis lucretur 30 imperiales. Quæritur quantum qualibet septimanâ expendere debeat, ut sibi ad finem anni supersint 500 Imperiales, unà cum expensa 4 septimanarum.

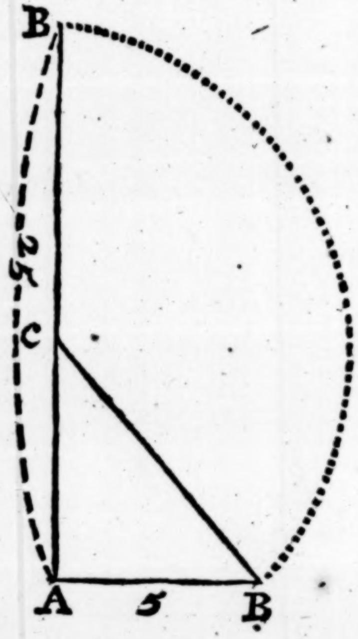
30. Operarius quidam exactis 40 ab opere incepto septimanis numerat in reposito 28 Imperiales — mercede 3 septimanarum; deprehenditque se expendisse 36 Imp. + mercede 11 septimanarum. Quæritur quantum singulis septimanis pro mercede acceperit, &c.

31. In

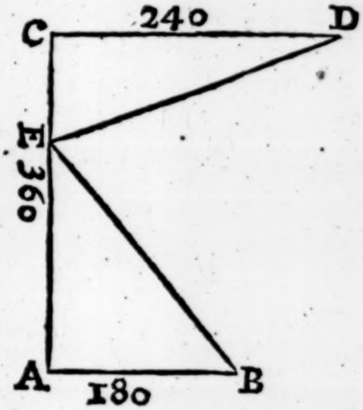



31. In Triangulo rectangulo ABC, datur basis $AB = 9$, & differentia reliquorum laterum, hoc est, Segmentum $BD = 3$. Quæruntur latera AC, BC.

32. In Triangulo rectangulo ABC, datur basis $AB = 5$, & summa reliquorum laterum $AC + BC = 25$. Quæruntur ipsa latera AC, BC, distinctim.




33. Sint

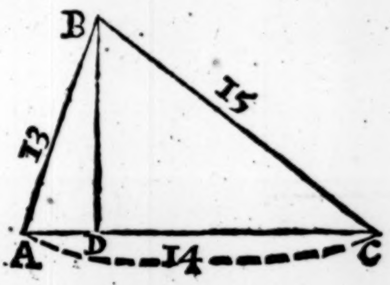



33. Sint duæ turres, AB altitudinis 180 pedum, & CD 240, in distantia AC 360 pedum. Applicare oporteat scalam super linea AC, puta in E, ejus longitudinis, ut inde ad summitatem utriusvis turris æque pertingat. Quæritur punctum E in linea distantiæ, ut & longitudo scalæ EB, ED.

§. *Hoc est, proponendo abstractè.*

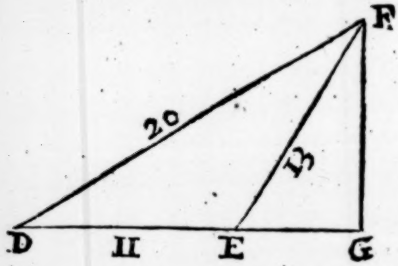
Duorum Triangulorum rectangulorum, æquales hypotenusas habentium, datur summa basium & cujusque altitudo. Quæruntur singulæ bases cum communi hypotenusâ.

34. In Triangulo ABC, dantur singula latera, AB=13, AC=14, BC=15; ductâque perpendiculari BD. Quæruntur segmenta baseos, AD, DC.

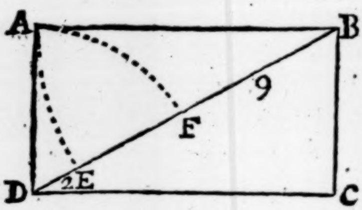



35. In

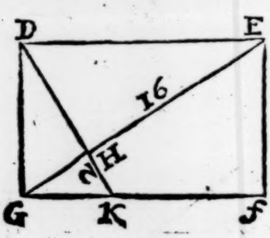
35. In triangulo obtusangulo DEF dantur singula latera, utpote DE 11, EF 13, DF 20; demissoque in basin prolongatam perpendiculo FG, Quæritur baseos prolongatio EG.



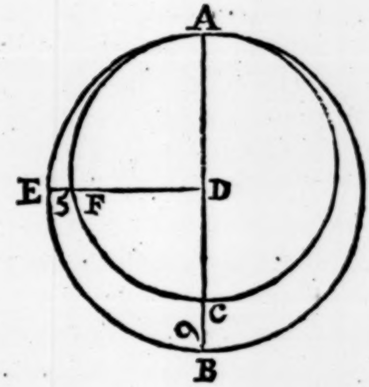

36. In rectangulo ABCD, datur differentia inter longitudinem AB, & diagonalem BD, hoc est DE=2; itemque differentia inter latitudinem AD & diagonalem BD, hoc est FB=9. Quæruntur rectanguli latera AB, AD.




37. In rectangulo DEFG, ex angulo D ad latus oppositum GF ducta est recta DK, intersecans diagonium EG ad angulos rectos in H: daturque segmentum HK=2, & HE =16. Quæruntur latera rectanguli.



38. Sit circulus, cujus diameter AB, quem alius minor, cujus diameter AC, interius tangat in A: erectoque ex D [centro majoris] super diametro AB, perpendiculariter radio DE, qui secet peripheriam minoris in F; detur BC [differentia diametrarum] =9, cum segmento EF = 5. Quæruntur circulorum dictorum Diametri AB, AC.




39. Duo sadoles habent aliquot aureos: quorum A dicit ad B; Si dederis mihi 1 aureum de tuis, totidem habebo ac tibi restabunt. Imò, respondet B, si tu mihi dederis 1 aureum de tuis, duplum habebo eorum qui tibi restabunt. Quæritur quot aureos quisque habuerit.

40. Quidam emit duos equos, cum Ephippio constante 100 Florenis: quod [Ephippium] si imponatur primo Equo [A] uterque equorum æquali pretii censebitur: sin vero imponatur alteri equo [B] dictum Ephippium, hic duplo erit pretiosior priori. Quæritur quanti constiterint dicti equi.

41. Oeno-

41. Oenopola quidam habet duplex vinum v.gr. A & B: quæ si misceantur ad partes æquales, Cantharus mixti constat 15 Stufris; si vero misceantur in ratione sesqui altera, puta si toties sumantur 2 Canthari de A, quoties 3 de B, Cantharus constabit 14 Stufris. Quæritur de pretio cujusque vini sigillatim.

42. Interroganti filio quot annos haberet, respondit pater, Si annis meis detraxeris 5, & reliquum diviseris per 8, quotus erit $\frac{1}{3}$ annorum tuorum: si vero annis tuis addideris 2, & totum multiplicaveris per 3, detractis ex producto 7, habebis ipsum numerum annorum meorum. Quæritur de ætate tam patris quam filii.

43. Reperire duos numeros, quorum summæ si addantur 6, totum sit duplum majoris numeri; & quorum differentiæ si detrahantur 2, quod remanet sit dimidium minoris.

44. Reperire duos numeros quorum productum sit 240, & triplum majoris divisum per minorem sit 5.

45. Duo volunt emere domum pro 1200 Florenis; dicitque A ad B, Si dederis mihi $\frac{2}{3}$ tui argenti, ego solus potero domum hanc redimere: cui B, si à te acceperim $\frac{3}{4}$ tuæ summæ, & ego solus redemero domum propositam. Quæritur quantum argenti quisque habuerit.

46. Juvenes

46. Juvenes aliquot & virgines debent hoſpiti pro exhibito convivio, 37 Florenos, ea ratione ut juvenum cuique aſſignentur 3 Floreni, virgini autem 2. Sique fuiſſent tot juvenes, quot fuerunt virgines, & è contra, ſervatâ eadem conditione, debuiſſent 4 Florenis minus quam nunc. Quæritur numerus juvenum ac virginum.

47. Dux bellicus poſt commiſſum prælium luſtrans exercitum, cujus peditatus fuerat triplus ipſius equitatus, deprehendit ante prælium aufugiſſe ex peditatu $\frac{1}{12}-120$, ex equitatu vero $\frac{1}{10}+120$, tum $\frac{1}{4}$ totius exercitus in præſidia diſtributos eſſe [connumeratis infirmis & vulneratis] ac ſupereſſe ſibi $\frac{3}{8}$ totius exercitus; reliquis qui deſiderantur, ipſo prælio extinctis aut captis; quorum [extinctorum] numero ſi addantur 3000, ſumma erit $=$dimidio peditum quos initio habuit. Quæruntur numeri ſingulorum.

48. Dividere numerum 100 bis in duas partes, ita ut major pars primæ diviſionis ſit tripla minoris partis ſecundæ diviſionis; & major ſecundæ ſit dupla minoris primæ.

49. Dividere numerum 30 bis in duas partes, ita ut major pars primæ diviſionis cum minori ſecundæ ſit 33; & ſumma partium minorum ſubducta ex ſumma majorum relinquat 14.

50. Maritus uxor & filius habent annos 96, ita ut mariti anni & filii ſimul faciant uxoris $+15$; ſed uxoris cum filii faciant annos mariti $+2$. Quæritur ætas ſingulorum.

51. Tres Mercatores ex diversis nundinis in diversorio convenientes, connumerant lucra sua, habentque summam 780 Flor. Porro si addatur lucrum primi & secundi, & ex aggregato subtrahatur lucrum tertii, remanet lucrum primi $+82$ Flor. sin autem addatur lucrum secundi & tertii, & ex aggregato subtrahatur lucrum primi, remanet lucrum tertii$-43$ Flor. Quæritur de lucro singulorum.

52. Tres personæ, A, B, C, debent certam pecuniæ summam, ita ut A cum B simul debeant 210 Florenos; B cum C, 290, & C cum A 400. Quæritur quantum unusquisque debeat.

53. Invenire tres numeros, ita ut primus cum dimidio reliquorum, secundus cum $\frac{1}{3}$ reliquorum & tertius cum $\frac{1}{4}$ reliquorum semper faciant 34.

54. Detur Quadratum in 9 quadratula distinctum: oportet invenire ac disponere numeros per singulas areolas, ita ut summa cujusque ordinis ternarii, sive lateraliter sive diagonaliter connumerando, semper sit 15.

55. [Theorema] Si dentur numeri quotcunque & qualescunque, fiatque subtractio minoris cujusque ex proxime majori: Dico summam istarum differentiarum æquari differentiæ numerorum maximi & minimi.

# DE

# Æquationib. Quadr.

## *Earumque Solutione.*

ÆQuationum Quadraticarum tres numerantur Casus seu Formulæ, ut sequitur:

### *Primus* Casus.

$$x^2 = ax + b^2$$

Solutio $\frac{1}{2}a$ mult.
$\frac{1}{2}a$

$$\frac{1}{4}a^2 + b^2$$

$$x = \sqrt{\tfrac{1}{4}a^2 + b^2} + \tfrac{1}{2}a = BD$$

### *Examen seu restitutio.*

$$x - \tfrac{1}{2}a = \sqrt{\tfrac{1}{4}a^2 + b^2}$$
$$x - \tfrac{1}{2}a$$

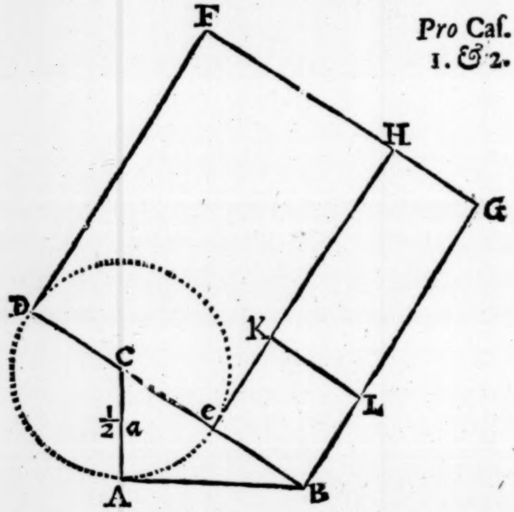
$$x^2 - ax + \tfrac{1}{4}a^2 = \tfrac{1}{4}a^2 + b^2$$
$$\therefore\ x^2 = ax + b^2$$

 Demonstratio

## *Demonstratio* Primi Casus.

Sit $AC=\frac{1}{2}a$, & $AB=b$: quibus conjunctis ad angulum rectum in A, describatur centro C, radio CA, Circulus AD*e*, ductâque recta DCB $=x$, fiat super eadem Quadratum $BDFG=x^2$; agatur porro *e*H, parallela ipsi BG.

*Pro* Cas. 1. & 2.




*Dico* 1.) $x^2$ esse $=ax+b^2$. Nam primum DC*e*$=a$, & $DF=x$, componunt rectang. $DH=ax$; deinde per 36.111 rectang. DB*e* [hoc est GB*e*] est=quadrat. AB, hoc est $b^2$. $\because ax+b^2=x^2$.

*Dico* 2.) $x$ esse $=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$. Nam Quadr. AC est $=\frac{1}{4}a^2$, & $\square AB=b^2$: quibus per 47.1 æquatur $\square BC$; ergo $BC=\sqrt{\frac{1}{4}a^2+b^2}$. adde $CD=CA=\frac{1}{2}a$ erit $BD=BC+CD$. Hoc est $x=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$. *q. e. d.*

*Secundus*

## *Secundus* CASUS.

$$x^2 = -ax + b^2$$

$$\begin{array}{ll} \text{Solutio} & -\tfrac{1}{2}a \ \text{mult.} \\ & -\tfrac{1}{2}a \\ \hline & +\tfrac{1}{4}a^2 + b^2 \end{array}$$

$$x = \sqrt{\tfrac{1}{4}a^2 + b^2} - \tfrac{1}{2}a = eB$$

## *Examen seu restitutio.*

$$\begin{array}{lrcl} & x + \tfrac{1}{2}a & = & \sqrt{\tfrac{1}{4}a^2 + b^2} \\ \text{mult.} & x + \tfrac{1}{2}a & & \\ \hline & x^2 + ax + \tfrac{1}{4}a^2 & = & \tfrac{1}{4}a^2 + b^2 \\ \therefore & x^2 & = & -ax + b^2 \end{array}$$

## *Demonstratio* Secundi Casus.

Constructâ figurâ ut ante, sit nunc $eB = x$; unde $DB = a + x$ descriptum autem ab $eB$ Quadratum $eL$, $= x^2$.

*Dico nunc* 1.) $x^2$ esse $= b^2 - ax$. Nam $GB = DB = a + x$, & $LB = eB = x$; ergo $GL = De = a$; quæ cum $KL = eB = x$, constituit ▭ $GK = ax$. Itaque ▭ $Ge$ [hoc est ▭ $GK + \square eL = x^2 + ax$]. Quod ipsum ▭ $Ge$, quum Casu præcedenti ostensum sit $= \square AB = b^2$; erit $x^2 + ax = b^2$ & $x^2 = b^2 - ax$.

*Dico* 2.) $x$ esse $= \sqrt{\frac{1}{4}a^2+b^2} - \frac{1}{2}a$. Nam Ce $=\frac{1}{2}a$, & $e$B$=x$, unde CB$=\frac{1}{2}a+x=\sqrt{\frac{1}{4}a^2+b^2}$ per Præced. ideóque $x=\sqrt{\frac{1}{4}a^2+b^2}-\frac{1}{2}a$. *q. e. d.*

---

## *Tertius* Casus.

$$x^2 = ax - b^2$$

Solutio $\frac{1}{2}a$ mult. $\frac{1}{2}a$

$$\frac{1}{4}a^2 - b^2$$

$x=\frac{1}{2}a+\sqrt{\frac{1}{4}a^2-b^2}$. Radix major, =BD

& $x=\frac{1}{2}a-\sqrt{\frac{1}{4}a^2-b^2}$. Radix minor, = BE

Habet enim tertius hic Casus duas veras radices: quarum nunc major, nunc minor, quinetiam sæpe utraque constructioni propositæ quæstionis inserviet prout usus docebit.

### *Examen seu restitutio.*

mult. $x-\frac{1}{2}a$ / $x-\frac{1}{2}a$ $= + \sqrt{\frac{1}{4}a^2-b^2}$

$$x^2-ax+\tfrac{1}{4}a^2 = \tfrac{1}{4}a^2-b^2$$
$$x^2-ax = -b^2$$
$$x^2 = ax-b^2$$

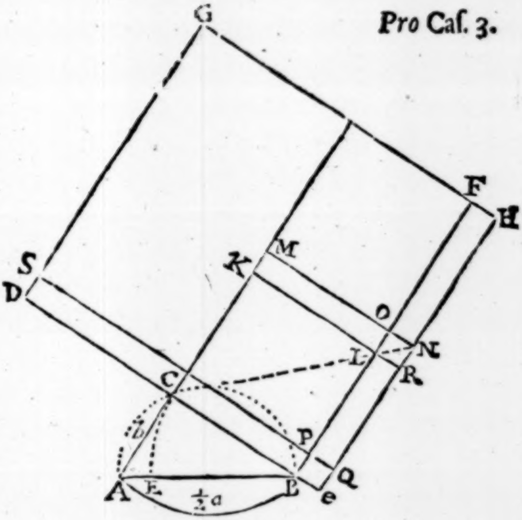
*Demonstratio*

## *Demonstratio* Tertii Casus.

Sit in figurâ sequenti, AB $= \frac{1}{2}a$, & AC$=$ $b$. Descripto super AB semicirculo, applicetur AC ad peripheriam, jungaturque BC: quâ prolongatâ fiat CD, itemque C$e$, $=$ AB.

Sit jam (1.) DB$= x$, tanquam radix major, super qua constitutum $\square$ DF erit $= x^2$. Agatur deinde $e$H parallela ipsi BF, occurrens GF productæ in H, erit ▭DH $= ax$.

*Dico nunc* 1.) $x$ esse $= \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2}$ per 47.1 & constructionem.




*Dico* 2.) $x^2$ esse $= ax - b^2$. Constituto enim super C$e = \frac{1}{2}a$ quadrato CN, erit illud $= \frac{1}{4}a^2$: itemque super CB$= \sqrt{\frac{1}{4}a^2 - b^2}$, descri-

pto quadrato CL, erit hoc $= \frac{1}{4}a^2 - b^2$. Quare gnomon KNB erit $= b^2$: sed gnomoni huic æquatur ▭ BH, quod sic ostenditur: Ductâ diametro CN, complementa ML, *e*L, sunt æqualia per 43.1 sed ipsi *e*L est etiam = ▭ OH: Si enim ex BF = BD auferatur BO = *e*N = *e*C, remanebit OF = BC, = BL: Unde cum & ▭ OH sit = OK, erit ▭ BH = gnomoni KNB $= b^2$; ac proinde □ DF = ▭ DH — ▭ BH; hoc est, $x^2 = ax - b^2$. *q. e. d.*

Sit vero (2) B*e*, = AE, = $x$, tanquam radix minor: Super qua constitutum quadratum BQ erit $= x^2$, & ▭ *e*S $= ax$: ex quo si auferatur □ BQ, erit reliquum ▭ BS = BH, $= b^2$. Unde jam reliqua facile patent.

Probl.

*Probl.* 56. Reperire numerum, quo multiplicato per 6, productoque subtracto ex quadrato ipsius numeri inveniendi, residuum sit 280.

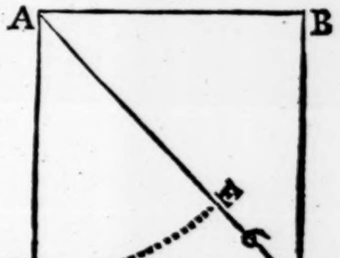
57. Reperire numerum quo multiplicato per 8, productoque addito ad Quadratum ipsius numeri inveniendi, aggregatum sit 660.

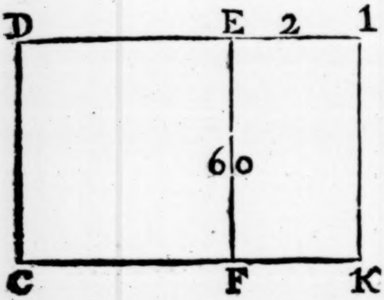
58. Dividere numerum 140 in duas partes, ità ut productum ipsarum partium sit = quadrato ex 56, hoc est 3136.

59. Sint 969 milites in aciem oblongam constituendi, ita ut differentia laterum majoris & minoris sit 40. Quæritur numerus militum cujusque ordinis, in longum & latum.

60. Sint iterum milites 480 in aciem oblongam disponendi, ita ut summa laterum majoris & minoris sit 52. Quæritur numerus militum cujusque ordinis in longum & latum.

61. In Quadrato ABCD datur differentia diametri & lateris, hoc est EC, = 6. Quæritur latus Quadrati.





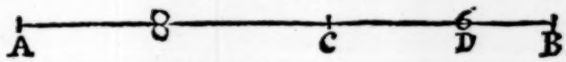

62. Quadrato DF additum est [sub eadem altitudine] rectangulum EK; cujus datur latitudo EL=2, nec non area totius rectanguli compositi DK, = 60. Quæritur latus Quadrati.

63. Quidam emit aliquot ulnas panni, pro 70 Florenis; deprehenditque si 4 ulnæ amplius fuissent, unamquamque ulnam tunc 2 Flor. viliùs emtam fuisse. Quæritur quot ulnas emerit.

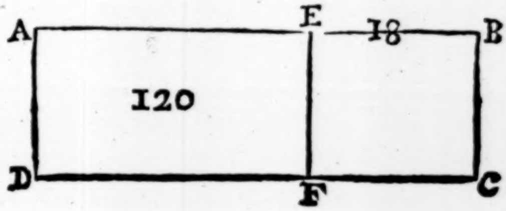
64. Aliquot sodales debent in diversorio summatim 175 Solidos: postquam vero, ante factam solutionem, duo ipsorum tacite excessere, symbola singulorum qui remanserant 10 Solidis adaucta deprehenduntur. Quæritur numerus sodalium.

65. Dividere numerum 21 in duas partes, quarum si major dividatur per minorem, & vicissim minor per majorem; tum prior quotiens multiplicetur per 4, & posterior pre 25, ut numeri producti sint æquales.

66. Sit




66. Sit linea AB secta in C, ita ut AC sit 8, CB vero 6 : oportet eandem lineam AB aliter secare, puta in D, ita ut rectangulum sub AD & DC sit æquale rectangulo sub AC & CB, seu producto ex 8 & 6, hoc est 48. Quæritur intersegmentum CD.



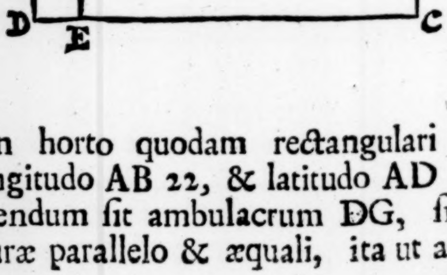

67. Sit hortus rectangularis ABCD, cujus longitudo AB tripla existat latitudinis AD : porro numeratis à B versus A 18 perticis, hoc est BE, ductâque EF parallela ipsi AD, detur area reliqui rectanguli ED 120 perticarum quadratarum. Quæritur dicti horti longitudo AB & latitudo AD.

68. Sint milites 600 in aciem oblongam dispositi : quam dum Colonellus latiorem cupit, deprehendit,

prehendit, si longitudini 10 ordines adimat, latitudini duos ordines accrescere. Quæritur numerus militum per singulos ordines in longum & latum.

69. Aliquis emit equum, quem rursus vendit pro 56 Coronatis, lucraturque tot coronatos ad 100, quot ipsi equus constiterat. Quæritur quanti emtus sit equus.

70. Quidam emit pro 30 Florenis lintea duplicis generis, puta subtilius & crassius: ulnam quidem subtilioris pro tot Florenis quot inde ulnas accepit: tum 28 ulnas crassioris, ita ut 8 ulnæ constent tot Florenis, quot una ulna subtilioris. Quæritur quot ulnas lintei subtilioris, & quo pretio utrumque emerit.




71. In horto quodam rectangulari ABCD, cujus longitudo AB 22, & latitudo AD 10, conconstituendum sit ambulacrum DG, situ ad latera figuræ parallelo & æquali, ita ut area dicti ambulacri

ambulacri seu gnomonis DG æquetur reliquo rectangulo FC [sive ut gnomon DG sit dimidia pars totius figuræ ABCD propositæ. Quæritur dicti gnomonis latitudo DE, BG.

72. Ex tribus numeris proportionalibus datur medius = 12, & differentia extremorum = 10. Quæruntur ipsi extremi.

73. Ex tribus numeris proportionalibus datur summa primi & secundi = 10, & differentia secundi & tertii = 24. Quæruntur singuli.

74. Ex quatuor numeris proportionalibus datur tertius = 12, tum summa primi & secundi = 8; porro si secundus numerus ex suo Quadrato subtrahatur, remanebit quartus. Quæruntur dicti numeri.

75. Ex quatuor numeris continue proportionalibus datur summa mediorum = 24, itemque summa extremorum = 56. Quæruntur singuli [supposito quod primus numerus sit minimus reliquorum.]

76. Duæ rusticæ A & B, ad forum simul adferunt 100 ova, vendendoque una tantundem accepit quam altera: ast A [quæ majora eóque meliora habuerat] dicit ad B, si æqualem tecum

ovorum

ovorum numerum habuissem, pro iis 18 Stufros accepissem; cui respondet B, Si paria numero tecum habuissem ova, accepissem pro iis tantummodo 8 Stufros. Quæritur quot ova unaquæque habuerit.

77. Duo rustici, A & B, vendunt frumentum pretio diverso: A quidem 20 Modios; B verò recepit pro uno Modio tot Florenos, quot Modios vendidit: deprehenditque A, si tot Modios vendidisset, quot B accepit Florenos, se tunc accepturum fuisse 252 Flor. quoniam vero ambo simul receperunt 176 Flor. Quæritur quot Modios A, & quo pretio B, vendiderit.

78. Duo mercatores panni vendunt ulnas 21: primus quidem 1 ulnam tot Coronatis, quanta est $\frac{1}{5}$ ulnarum secundi; secundus autem 1 ulnam tot Coronatis, quanta est $\frac{1}{3}$ vlnarum primi: peractâque venditione connumerant 48 Coronatos. Quæritur quot ulnas singuli & quo pretio vendiderint.

79. Duorum mercatorum serici primus 40 ulnas habet, secundus 90: primus Coronato vendit $\frac{1}{3}$ ulnæ plus quam secundus: tum venditione peractâ ambo numerant Coronatos 42. Quæritur quot ulnas quisque Coronato vendiderit.

80. Reperire

80. Reperire numerum, cujus quadruplo si addantur 91, totum sit ad Quadratum ipsius numeri quæsiti ut 3 ad 4.

81. Reperire numerum à cujus duplo si subtrahantur 12, Quadratum reliqui minus 1, sit noncuplum numeri quæsiti.

82. Dividere numerum 19 in duas partes, ita ut summa Quadratorum à partibus sit 193.

83. Dividere 7 in duas partes ita ut differentia Quadratorum quæ fiunt à triplo partis minoris & à duplo partis majoris, sit 17.

84. Quidam emit volumen lintei, quod rursus vendendo lucratur 12 Florenos minus $\frac{1}{10}$ expensi pretii: Deprehenditque se hac ratione tantundem lucratum esse pro 100, quanti ipsi constiterat linteum. Quæritur quanti emtum sit linteum venditumque.

85. Quidam emit 18 ulnas panni diversi generis ac coloris, puta rubri & nigri; de unoquoque pro Florenis 40: solvitque pro singulis ulnis rubri 1 Flor. plus quam pro nigro. Quæritur quot ulnas cujusque generis emerit.

86. Quidam

86. Quidam emit 120 libras piperis, totidemque zingiberis: ita ut 1 librâ plus zingiberis pro floreno acceperit quam piperis: adeoque totum pretium piperis 6 Florenis superet pretium zingiberis. Quæritur quot libras de unoquoque pro Floreno acceperit.

87. Quidam emit 80 libras piperis, & 36 libr. croci, ita ut pro 8 Flor. acceperit 14 libras piperis plus quam croci pro 26 Flor. estque summa expensi pretii 188 Flor. Quæritur quot libras piperis pro 8 Flor. & quot libras croci pro 26 Florenis acceperit.

88. A & B debent simul 174 florenos: & A quidem singulis diebus persolvit 8 Flor. B vero primo die 1 Flor. secundo 2, tertio 3, & sic consequenter. Quæritur quot diebus totum debitum persolvant; & quantum unusquisque debeat.

89. Quidam peregrinari instituit tot dies quot coronatos habebat: accidit autem ut ipsi singulis sequentibus itineris diebus, totidem ac præcedenti die habuerat, & insuper bini accesserint coronati; absolutoque ut decreverat itinere, deprehendit se omnino 45 coronatos habuisse. Quær. quot initio habuerit.

90. Viator

90. Viator quidam quotidie 9 milliaria conficit, hunc triduo post alter insequitur, qui primo die 4 milliar. conficit, secundo 5, tertio 6, & sic deinceps, singulis diebus 1 milliare lucrando. Quæritur quando hic priorem assequetur.

91. Ex urbibus A & B, quarum distantia 70 milliarium, eodem tempore exit viator, quorum unus singulis diebus conficit milliaria 6; alter vero primo die 2, secundo $2\frac{1}{2}$ tertio 3, & ita deinceps cujusque diei itineri $\frac{1}{2}$ milliare adjiciendo. Quæritur quando sibi mutuo obviabunt, &c.

92. Iterum, ex duabus urbibus A & B, quarum distantia sit 120 milliarium, eodem tempore exit viator, quorum primus quotidie conficit 5 milliaria, alter vero quotidie 3 milliar. minus quam est numerus dierum quibus conveniunt. Quæritur ergo quando id fiat.

93. Ex loco A versus B mittitur tabellarius qui singulis diebus 8 milliaria conficit: huic post 27 milliaria confecta alter ex B obviam mittitur, qui quotidie $\frac{1}{20}$ totius viæ seu intervalli locorum A & B conficit, totidemque diebus [quanta nimirum est $\frac{1}{20}$ dictæ distantiæ] completis priori occurrit. Quæritur quantum distent loca A & B.

94. Duo

94. Duo mercatores A & B faciunt societatem, in quam B quidem confert 420 Flor. A vero recipit ex lucro 52 Flor. estque summa sortis utriusque unà cum lucro 854 Flor. Quæritur quantum contulerit A, & quantum ex lucro receperit B.

95. Filius interrogat patrem de ætate sua; cui pater: Si annis meis demantur 4, reliquum erit triplum annorum tuorum: Si vero ex annis tuis tollatur 1, dimidium reliqui erit radix quadrata annorum meorum. Quær. ætas tam patris quam filii.

96. Reperire duos numeros, quorum summa Quadratorum sit 317, & productum si invicem multiplicentur, sit 154.

97. Reperire duos numeros, quorum productum sit 108, & differentia Quadratorum 63.

98. Duo rustici vendunt frumentum diversi generis: A quidem 6 Modios; B vero recipit pro suo summatim 20 Florenos: dicitque B ad A: Ecce si addamus numeros meorum Modiorum & tuorum Florenorum, summa erit 28: resondet A, Et si addam Quadratum meorum Florenorum Quadrato tuorum Modiorum, aggregatum erit 424. Quær. quot Modios vendiderit B, & quot Florenos receperit A.

99. Repe-

99. Reperire duos numeros, quorum primus +2, multiplicatus in secundum −3, producat 110: & vicissim primus −3, multiplicatus in secundum +2, producat 80.

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## *Tabulæ Poteſtatum Numericarum.*

| √ | Qua. | √ | Qua. | √ | Quad. | √ | Quad. | √ | Quadr. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 21 | 441 | 41 | 1681 | 61 | 3721 | 81 | 6561 |
| 2 | 4 | 22 | 484 | 42 | 1764 | 62 | 3844 | 82 | 6724 |
| 3 | 9 | 23 | 529 | 43 | 1849 | 63 | 3969 | 83 | 6889 |
| 4 | 16 | 24 | 576 | 44 | 1936 | 64 | 4096 | 84 | 7056 |
| 5 | 25 | 25 | 625 | 45 | 2025 | 65 | 4225 | 85 | 7225 |
| 6 | 36 | 26 | 676 | 46 | 2116 | 66 | 4356 | 86 | 7396 |
| 7 | 49 | 27 | 729 | 47 | 2209 | 67 | 4489 | 87 | 7569 |
| 8 | 64 | 28 | 784 | 48 | 2304 | 68 | 4624 | 88 | 7744 |
| 9 | 81 | 29 | 841 | 49 | 2401 | 69 | 4761 | 89 | 7921 |
| 10 | 100 | 30 | 900 | 50 | 2500 | 70 | 4900 | 90 | 8100 |
| 11 | 121 | 31 | 961 | 51 | 2601 | 71 | 5041 | 91 | 8281 |
| 12 | 144 | 32 | 1024 | 52 | 2704 | 72 | 5184 | 92 | 8464 |
| 13 | 169 | 33 | 1089 | 53 | 2809 | 73 | 5329 | 93 | 8649 |
| 14 | 196 | 34 | 1156 | 54 | 2916 | 74 | 5476 | 94 | 8836 |
| 15 | 225 | 35 | 1225 | 55 | 3025 | 75 | 5625 | 95 | 9025 |
| 16 | 256 | 36 | 1296 | 56 | 3136 | 76 | 5776 | 96 | 9216 |
| 17 | 289 | 37 | 1369 | 57 | 3249 | 77 | 5929 | 97 | 9409 |
| 18 | 324 | 38 | 1444 | 58 | 3364 | 78 | 6084 | 98 | 9604 |
| 19 | 361 | 39 | 1521 | 59 | 3481 | 79 | 6241 | 99 | 9801 |
| 20 | 400 | 40 | 1600 | 60 | 3600 | 80 | 6400 | 100 | 10000 |

| √c. | Cub. | √c. | Cub. | √c. | Cub. | v√ | Q-qu | v√ | Q-qua. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 11 | 1331 | 21 | 9261 | 1 | 1 | 11 | 14641 |
| 2 | 8 | 12 | 1728 | 22 | 10648 | 2 | 16 | 12 | 20736 |
| 3 | 27 | 13 | 2197 | 23 | 12167 | 3 | 81 | 13 | 28561 |
| 4 | 64 | 14 | 2744 | 24 | 13824 | 4 | 256 | 14 | 38416 |
| 5 | 125 | 15 | 3357 | 25 | 15625 | 5 | 625 | 15 | 50625 |
| 6 | 216 | 16 | 4096 | 26 | 17576 | 6 | 1296 | 16 | 65536 |
| 7 | 343 | 17 | 4913 | 27 | 19683 | 7 | 2401 | 17 | 83521 |
| 8 | 512 | 18 | 5832 | 28 | 21952 | 8 | 4096 | 18 | 104976 |
| 9 | 729 | 19 | 6859 | 29 | 24389 | 9 | 6561 | 19 | 130321 |
| 10 | 1000 | 20 | 8000 | 30 | 27000 | 10 | 10000 | 20 | 160000 |

*Tabulæ*

## *Tab. reductoriæ Numerorum irrationalium.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| √6 = | 2√1½ | √75 = | 5√3 | √200 = | 5√8 | √486 = | 9√6 |
| √8 | 2√2 | √80 | 4√5 | √216 | 6√6 | √490 | 7√10 |
| √10 | 2√2½ | √90 | 3√10 | √243 | 9√3 | √500 | 10√5 |
| √12 | 2√3 | √96 | 4√6 | √245 | 7√5 | √512 | 8√8 |
| √18 | 3√2 | √98 | 7√2 | √250 | 5√10 | √539 | 7√11 |
| √20 | 2√5 | √99 | 3√11 | √252 | 6√7 | √567 | 9√7 |
| √24 | 2√6 | √108 | 6√3 | √275 | 5√11 | √600 | 10√6 |
| √27 | 3√3 | √112 | 4√7 | √288 | 6√8 | √640 | 8√10 |
| √28 | 2√7 | √125 | 5√5 | √294 | 7√6 | √648 | 9√8 |
| √32 = | 4√2 | √128 = | 8√2 | √300 = | 10√3 | √700 = | 10√7 |
| √40 | 2√10 | √147 | 7√3 | √320 | 8√5 | √704 | 8√11 |
| √44 | 2√11 | √150 | 5√6 | √343 | 7√7 | √800 | 10√8 |
| √45 | 3√5 | √160 | 4√10 | √360 | 6√10 | √810 | 9√10 |
| √48 | 4√3 | √162 | 9√2 | √384 | 8√6 | √891 | 9√11 |
| √50 | 5√2 | √175 | 5√7 | √392 | 7√8 | √1000 | 10√10 |
| √54 | 3√6 | √176 | 4√11 | √396 | 6√11 | √1100 | 10√11 |
| √63 | 3√7 | √180 | 6√5 | √405 | 9√5 | √1200 | 10√12 |
| √72 | 3√8 | √192 | 8√3 | √448 | 8√7 | | |
| 2√2 = | √8 | 2√5 = | √20 | 2√7 = | √28 | 2√10 = | √40 |
| 3√2 | √18 | 3√5 | √45 | 3√7 | √63 | 3√10 | √90 |
| 4√2 | √32 | 4√5 | √80 | 4√7 | √112 | 4√10 | √160 |
| 5√2 | √50 | 5√5 | √125 | 5√7 | √175 | 5√10 | √250 |
| 6√2 | √72 | 6√5 | √180 | 6√7 | √252 | 6√10 | √360 |
| 7√2 | √98 | 7√5 | √245 | 7√7 | √343 | 7√10 | √490 |
| 8√2 | √128 | 8√5 | √320 | 8√7 | √448 | 8√10 | √640 |
| 9√2 | √162 | 9√5 | √405 | 9√7 | √567 | 9√10 | √810 |
| 10√2 | √200 | 10√5 | √500 | 10√7 | √700 | 10√10 | √1000 |
| 2√3 = | √12 | 2√6 = | √24 | 2√8 = | √32 | 2√11 = | √44 |
| 3√3 | √27 | 3√6 | √54 | 3√8 | √72 | 3√11 | √99 |
| 4√3 | √48 | 4√6 | √96 | 4√8 | √128 | 4√11 | √176 |
| 5√3 | √75 | 5√6 | √150 | 5√8 | √200 | 5√11 | √275 |
| 6√3 | √108 | 6√6 | √216 | 6√8 | √288 | 6√11 | √396 |
| 7√3 | √147 | 7√6 | √294 | 7√8 | √392 | 7√11 | √539 |
| 8√3 | √192 | 8√6 | √384 | 8√8 | √512 | 8√11 | √704 |
| 9√3 | √243 | 9√6 | √486 | 9√8 | √648 | 9√11 | √891 |
| 10√3 | √300 | 10√6 | √600 | 10√8 | √800 | 10√11 | √1100 |
| 2√c.2 = | √c.16 | 2√c.3 = | √c.34 | 2√c.4 = | √c.32 | 2√c.5 = | √c.40 |
| 3√c.2 | 54 | 3√c.3 | 81 | 3√c.4 | 108 | 3√c.5 | 135 |
| 4√c.2 | 128 | 4√c.3 | 192 | 4√c.4 | 256 | 4√c.5 | 320 |
| 5√c.2 | 250 | 5√c.3 | 375 | 5√c.4 | 500 | 5√c.5 | 625 |
| 6√c.2 | 432 | 6√c.3 | 648 | 6√c.4 | 864 | 6√c.5 | 1080 |
| 7√c.2 | 686 | 7√c.3 | 1029 | 7√c.4 | 1372 | 7√c.5 | 1715 |
| 8√c.2 | 1024 | 8√c.3 | 1536 | 8√c.4 | 2048 | 8√c.5 | 2560 |
| 9√c.2 | 1458 | 9√c.3 | 2187 | 9√c.4 | 2916 | 9√c.5 | 3645 |
| 10√c.2 | 2000 | 10√c.3 | 3000 | 10√c.4 | 4000 | 10√c.5 | 5000 |

# LOGISTICA LINEARIS

## Pro Conſtructione *Æquationum Geometricæ*.

### PROPOSITIO I.

*Lineæ rectæ datæ datam rectam addere earumque exhibere ſummam.*

SINT addendæ $a$ & $b$; ſumma erit $a+b=d$.

### PROP. II.

*Ex data recta majore minorem ſubtrahere earumque exhibere differentiam.*

SUbtrahatur $c$ ex $b$; differentia erit $b-c=f$.

PROP.

## Prop. III.

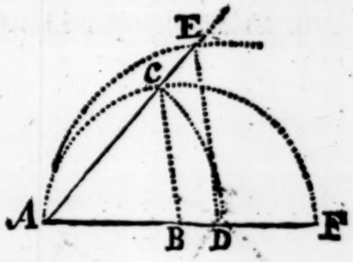
*Datis duabus rectis tertiam proportionalem invenire [idq; quàm fieri possit summa exactitudine.]*

### Casus 1.

*Si datarum differentia sit mediocris, seu non nimis vasta.*

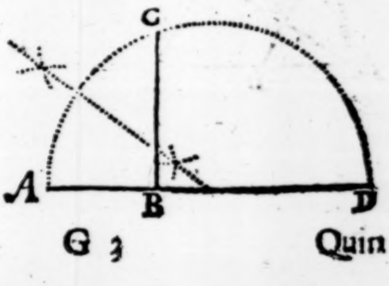
INtervallo primæ AB describatur ex B semicirculus ACF, in quo ex A accommodetur secunda AC, eaque ultra C infinite producatur: tum eadem secunda ex A collocetur in Diametrum semicirculi, puta in D, atque ex D, intervallo DA descriptâ peripheriâ, in productâ AC dabit punctum E, eritque AE tertia proportionalis quæsita.

AB $= a$, prima.
AC, AD $= b$, secunda.
AE $= \frac{b^2}{a}$, tertia quæs.




*Vel etiam ità.*

AB, prima.
BC, secunda.
BD, tertia quæsit.

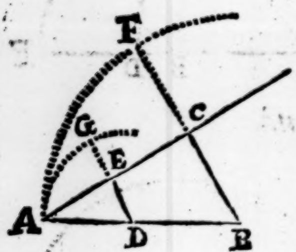


Quin circa primum hunc Caſum, ubi ſcil. datarum differentia non eſt juſto major, variæ operationes mediæque commode adhiberi poſſunt [ uti & in ſubſeq. *Prop.* IV. ] at non pariter in cœteris Caſibus.

Caſus 2.

*Si datarum differentia ſit nimis magna; & prima major exiſtat quam altera.*

Intervallo primæ AB, & B fiat angulus 60 grad. ABF; è cujus altero crure BF abſcindatur ſecunda BC, ducaturque AC. Fiat nunc AD= ſecundæ BC, atque ex D, intervallo DA [vel quolibet alio] deſcribatur angulus itidem 60 grad. ADG, cujus alterum crus DG à recta AC ſecabitur in E, eritque DE tertia proportionalis quæſita.




AB $= a$ prima.

$\left.\begin{matrix}\text{BC}\\\text{AD}\end{matrix}\right\} = b$ ſecunda.

DE $= \frac{b^2}{a}$ tertia quæſita.

Caſus

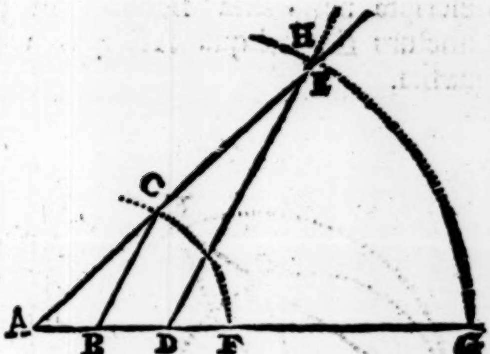
Casus 3.

*Si datarum differentia sit nimis magna, & prima minor existat quam altera.*

1. Ex B, termino primæ, intervallo secundæ BC, designetur angulus 60 grad. BFC, & ex A per C agatur recta infinita. 2. Ex D termino secundæ, intervallo satis magno designetur iterum angulus 60 grad. GDH, ductâque rectâ usque dum occurrat infinitæ AC in E, erit DE tertia proportionalis quæsita.

Cætera ad hanc Prop. observanda vid. ad subseq.

AB $= a$, pri.
BC
AD $= b$, sec.
DE $= \frac{b^2}{a}$, ter. quæsita.



## Prop. IV.

*Datis tribus rectis quartam proportionalem invenire.*

### Casus I.

*Si datarum [puta primæ & alterutrius reliquarum] differentia non sit immodica, & præsertim si prima minor existat altera reliquarum:*

INtervallo primæ AB, ex B describatur semicirculus ACF, in quo ex A accommodetur secunda AC, eaque ulterius infinitæ producatur: tum tertia ex A collocetur in Diametrum semicirculi, puta in D; atque ex D, intervallo DA, descripta peripheria signabit in producta AC punctum E, eritque AE quarta proportionalis quæsita.


AB $=$ $a$ prima
AC $=$ $b$ secunda
AD $=$ $c$ tertia
AE $=$ $\frac{bc}{a}$ quarta quæsita.

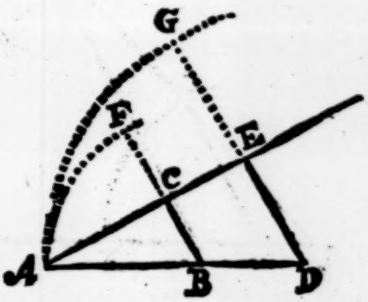
Casus

## Caſus 2.

*Si datarum [primæ & utriuſlibet aut alterutrius reliquarum] differentia ſit immodica; & prima major exiſtat alterutra vel utravis reliquarum.*

Intervallo primæ AB, ex B fiat angulus 60 grad. ABF, è cujus altero crure BF abſcindatur ſecunda BC, ducaturque ex A per C recta infinita: Collocetur jam tertia ex A in D; & ex D, intervallo DA [vel quolibet alio] fiat itidem angulus 60 grad. ADG, cujus alterum crus DG ſecabit productam AC, eritque DE quarta proportionalis quæſita.

AB $= a$, prima
BC $= b$, ſecunda
AD $= c$, tertia
DE $= \frac{bc}{a}$ quarta quæſita.



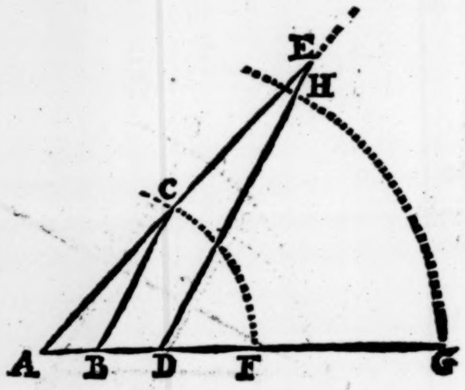

[*Nota*, Si prima non ſit major utrâque reliquarum, ſed duntaxat alterutrâ, uti hoc loco, operatio commode etiam inſtituetur juxta *Caſ.* 1. ſecundâ nimirum & tertiâ invicem permutalis.]

Caſus

## Caſus 3.

*Si datarum* [ primæ & utriuſlibet reliquarum ] *differentia ſit immodica; & prima exiſtat minor.*

1. Ex B, termino primæ, intervallo ſecundæ BC, deſignetur angulus 60 grad. BFC; & ex A per C agatur recta infinita. 2. Ex D, termino tertiæ, intervallo ſatis magno deſignetur iterum angulus 60 grad. GDH, ductâque recta DH, uſque dum occurrat infinitæ AC in E, erit DE quarta proportionalis quæſita.

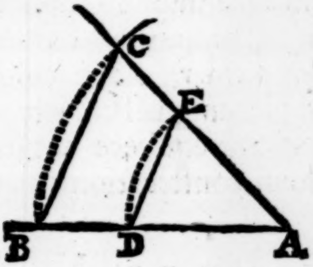



AB $=a$, prim.
BC $=b$, ſec.
AD $=c$, tertia
DE $=\frac{bc}{a}$ quar. quæſ.

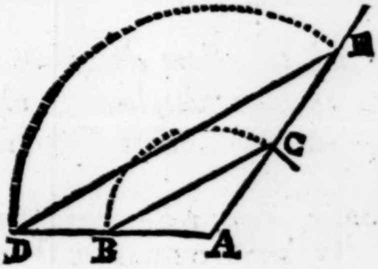
Vel, aſſumatur dupla primæ, & prodibit dimidia quæſitæ: ac tunc ſæpe operatio ad *Caſum* 1. reduci poterit.

§. Cæterum niſi prima datarum ſit juſto brevior, praxis generatim [ hujuſce & præced. propoſ. ] non minus accurata quam expedita eſt quæ fit per triangulum æquicrurum uti vulgò notum.

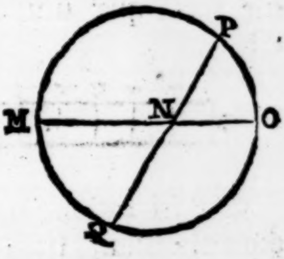
AB Prima




AB prima
BC ſecunda
AD tertia
DE quarta quæſ.




§. Interim notari quoque poſſunt modi ſequentes ex 35. & 36. III. *Euclid.*


FK prima
FG ſecunda
FH tertia
FL quarta

NQ prima
MN ſecunda
NO tertia
NP quarta quæſita.

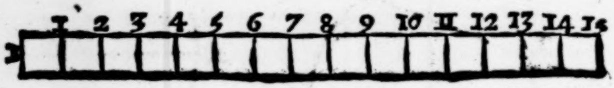
§. *Observandum* tamen, non quemlibet modorum recensitorum cuilibet Problemati construendo, quantum ad demonstrationem concinnandam attinet, æquè commodè aut rectè adhiberi, sed plerumque Problemata suam quodque constructionem requirere aut secum ferre; at nobis hic saltem constructionis exactitudo curæ esse debuit.

## Prop. V. [Lemma.]

*Lineam rectam datam in rectam alteram ducere seu multiplicare, ut productum sit quasi linea.* Seu Geometricè loquendo.

*Super data recta rectangulum constituere; quod* [valore suo numerico] *æquipolleat ipsi lineæ propositæ.*

Fiet hoc si recta multiplicans sit unitas, seu si pro altero latere constituendi plani assumatur recta quæ unitatem referat, uti hic videre est.




§. Pari ratione constituetur quantitas quolibet dimensionum, quæ tamen unicam duntaxat dimensionem repræsentet.

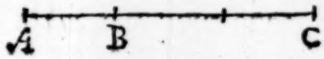
Sic etiam quælibet quantitas in lineam multiplicabitur; ut productum æquipolleat quantitati datæ.

PROP.

## PROP. VI.

*Datam rectam per numerum multiplicare.*

UT si data recta AB multiplicanda sit per 3 prodibit recta AC.




## PROP. VII.

*Datum planum per numerum multiplicare.*

UT si datum planum A*c* multiplicandum sit per 5, prodibit planum A*e*.


## PROP. VIII.

*Datum planum per lineam dividere.*

[Quod est fractionem in proportionem resolvere.]

SIT ab ▭ dividendum per *c*, fiet, ut *c* divisor ad *a*: Ita *b* ad $\frac{ab}{c}$ seu quotum quæsit. $=x$.

*Nota* :

*Nota:* Si planum dividendum ſit compoſitum, ut $\frac{a^2+b^2}{c}$, $\frac{c^2+b^2-ab}{2a}$, &c.

## PROP. IX.

*Datam lineam rectam per rectam dividere.*

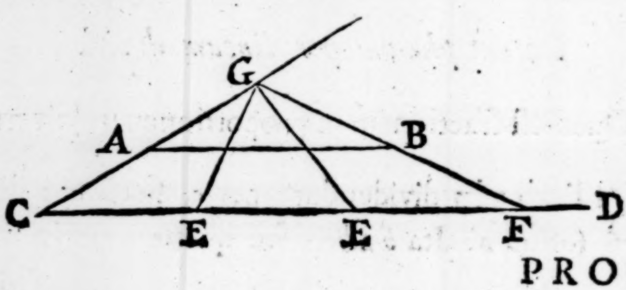
COgitetur ſuper data recta conſtitutum eſſe planum, cujus alterum latus ſit unitas [per V. præced.] atque tunc fiat juxta VIII. præced.

Ut ſi dividere oporteat $b$ per $a$; fiet ut $a$ ad $b$; ita unitas $= e$ ad $\frac{eb}{a} = \frac{b}{a}$.

## PROP. X.

*Datam rectam per numerum dividere.*

UT ſi dividenda ſit recta AB per 3, ſive pars tertia ipſius exhibenda. Ducatur CD parallela datæ AB; dein quovis intervallo fiant CE, EE, & EF æquales inter ſe & numero dato. Demum jungantur CA & FB donec coeant ut ad G. Poſtremò junge GE, EE, &c. & habebis lineam diviſam prout oportet.

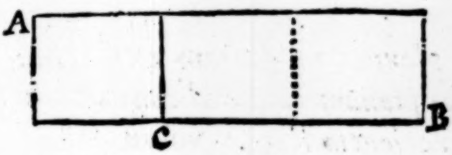



PROP.

## Prop. XI.

*Datum planum per numerum dividere.*

SIT propoſitum planum AB dividendum per 3, Quotus erit tertia pars plani propoſiti, puta AC.



## Prop. XII.

*Datum planum in aliud tranſmutare quod habeat datum latus ;* ſeu *datum planum ad datam longitudinem vel latitudinem reducere.*

SIT ▭ *bc* tranſmutandum in aliud, cujus unum latus ſit = datæ rectæ *d*; ſeu ▭ *bc* ſit reducendum ad longitudinem *d*, quæritur latitudo.

Dividatur [per VIII. præced.] propoſitum ▭ *bc* per datum latus *d*, & prodibit alterum latus quæſitum; hoc eſt, fiat ut *d* ad *b*, ita *c* ad quartam = *f*, = lateri quæſito, unde jam *df* = *bc*.

§. Patet hinc, qua ratione quælibet quantitas quotcunque dimenſionum poſſit reduci ad datas literas, ſeu ad datam literam [per unum] magnitudinem. Sit

Sit quantitas $acb^2$ ita tranſmutanda, ut in ea ter reperiatur litera $a$ [ſeu $a$ cubicum]. Aſſumpto itaque primum quadrato $b^2$, fiet ut $a$ ad $b$, ita $b$ ad tertiam $= d$; unde ad $= b^2$, & $acad$ ſeu $aacd$ $= acb^2$. Tum porro aſſumpto quadrato $cd$ fiet ut $a$ ad $c$, ita $d$ ad quartam $= f$; unde $af = cd$, & $aaaf$ ſeu $a^3f = acb^2$, quod deſiderabatur.

## Prop. XIII.

*Datum planum in lineam exponere: ſeu dadatum planum in aliud convertere quod habeat rationem lineæ* [vel ut vulgo effertur] *Lineam in lineam multiplicare juxta datam unitatem.*

DIvidatur propoſitum planum per unitatem, Quotus dabit quæſitum. Sit Quadrat. $ab$ exponendum in lineam, fiet, ut unitas, $=e$, ad $a$, ita $b$ ad quartam $=f$; unde $ef = ab$, & $ef$ quoque $=$ ipſi lineæ $f$, per ✓ præced.

§. Patet hinc qua ratione quælibet quotcunque dimenſionum quantitas in lineam exponi, ſed ad valorem linearum redigi poſſit. Sit ex. gr. quantitas $a^2b^2$ exponenda in lineam.

Suppoſitâ unitate $= e$, aſſumptoque primum Quadrato $a^2$, fiet; ut $e$ ad $a$, ità $a$ ad tertiam $=d$, unde $ed = a^2$. Similiter aſſumpto Quadrato $b^2$, fiet; ut $e$ ad $b$, ità $b$ ad tertiam $=f$, unde $ef = b^2$, & $eedf = a^2b^2$: itaque tandem aſſumpto Quadrato $df$ fiet: ut $e$ ad $d$, ità $f$ ad quartam $=g$, unde $eg = df$; & conſequentèr $e^3g = a^2b^2$; & ſimul

mul etiam $e^3g =$ ipſi $g$ numericè; quod expetebatur.

§. Quod ſi vero in propoſito exemplo [$a^2b^2$] ipſa quantitas $a$ ſumatur pro unitate, brevius ad ſcopum venietur; nimirum reducendo duntaxat Quadrat. $b^2$ ut ſequitur: ut $a$ ad $b$, ita $b$ ad tertiam $= k$; unde $ak = b^2$; ac proinde $a^3k = a^2b^2$ $=$ ipſi $k$.

## Prop. XIV.

*Datam quantitatem quamcunque per quamlibet aliam dividere.* [ſeu *Fractionem quamcunque in proportionem reſolvere.*]

### Caſus 1.

*Cum Diviſor pauciores dimenſiones habet quam dividendus.* Quæ eſt Diviſio ordinaria, ut $\frac{bd^2}{ac}$, $\frac{a^2df}{bg}$, $\frac{a^2b^2+bf^3}{abf-af^2}$.

### Caſus 2.

*Cum Diviſor & dividendus dimenſionibus pares exiſtunt,* ut $\frac{ab}{c^2}$, $\frac{d^2f}{ab^2}$, $\frac{b^2ac}{f^2d^2}$.

Tunc in dividendo adſciſcenda eſt unitas per V. præced. quæ ſit ex. gr. $= e$; erit igitur $\frac{eab}{c^3} = \frac{ab}{c^2}$ propoſito; & ſic etiam in cæteris.

## Caſus 3.

*Cum Diviſor plures dimenſiones habet quam dividendus, ut* $\frac{b}{a^2}$, $\frac{f^2}{cd^2}$, $\frac{ac^2}{b^2 f^3}$.

Tunc in dividendo adſciſcenda eſt unitas quoties opus, puta uſque dum ipſe dividendus numero dimenſionum ſuperet diviſorem. Suppoſita igitur unitate $= e$ ut ante, ponetur

$$\frac{e^2 b}{a^2} = \frac{b}{a^2}, \quad \frac{e^2 f^2}{cd^2} = \frac{f^2}{cd^2}, \text{ \&c.}$$

## Caſus 4.

*Cum Diviſor pauciores dimenſiones habet quam ratio ſeu ſpecies quoti exigit.*

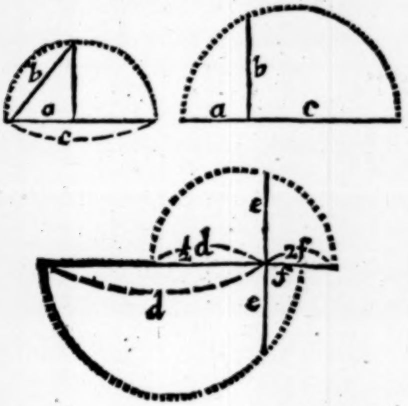
Ut ſi propoſitum ſit $\frac{ab^2}{c}$, & quotus debeat eſſe linea. Tunc in diviſore adſciſcenda eſt unitas, ut ſit $\frac{ab^2}{ec} = \frac{ab^2}{c}$, &c.

## Prop. XV.

*Datis duabus rectis mediam proportionalem invenire.*

DEntur $a$ & $c$; prodibit media $= b$, per conſtructiones vulgò notas,

Quodſi




Quodsi alterutra datarum sit nimis parva, sumatur ejusdem dupla vel tripla, & alterius dimidia vel tertia pars, proveniet enim ipsa quæsita, ut ante.

Dentur exemp. gr. *d* & *f*, media erit *e*, utroque modo reperta, ut figura ostendit.

## PROP. XVI.

*Dati plani* [solitarii] *latus quadratum reperire.*

MEdia proportionalis inter propositi plani [puta rectanguli] longitudinem & latitudinem exhibet latus quadratum quæsitum.

## PROP. XVII.

*Dati plani plano aucti vel diminuti latus quadratum invenire.*

UT si dentur $a^2+b^2$, $a^2-b^2$, $a^2+b^2-c^2$, &c. Constructio ex 47. I. & 31. III. patet.

## PROP. XVIII.

*Dati plani per numerum multiplicati vel diviſi latus quadratum reperire.*

UT ſi dentur $3a^2$, $\frac{1}{2}b^2$, $\frac{2}{3}ab$, &c.
Conſtructio plerumque duplici ratione perfici poteſt per 47. I. & 13. VI.

## PROP. XIX.

*Dati plani tam plano quam numero affecti* [juxta præced. XVII, XVIII.] *latus quadratum invenire.*

UT ſi dentur $2a^2 + \frac{1}{3}b^2$, $5b^2 + d^2 - \frac{1}{2}c^2$, &c.
Conſtructio patet ex præcedentibus.

## PROP. XX.

*Datæ quantitatis cujuſcunque, ad duas dimenſiones diviſæ latus quadratum invenire.*

UT ſi detur $\frac{ab^2}{c}$ $\frac{a^2d^2}{bc}$ $\frac{c^2f^2+d^4}{c^4-d^2}$ &c. Fiat $b$ ad $q$, $c$ ad $a$: deinde medium proportionale inter $b$ & $q$ erit latus quæſitum. Et ſic de cæteris.

PROP.

## Prop. XXI.

*Ex data linea recta radicem quadratam extrahere.*

INtelligatur super data recta constitutum esse planum cujus alterum latus sit unitas [quæ quidem interdum vera seu determinata requiritur; interdum autem quævis hypothetica ac pro lubitu assumpta eidem quæsito satisfaciet] ac tum [juxta XV. & XVI.] media proportionalis inter dictam unitatem & rectam propositam dabit radicem quæsitam.

Quod si data unitas sit nimis parva, assumetur ejus duplum vel triplum, & è contra propositæ lineæ dimidium vel trieus, prout ad Propos. XV. monitum fuit.

## Prop. XXII.

*Ex data quantitate trium dimensionum radicem quadratam extrahere.*

1. SIt quærenda $\sqrt{a^3}$, quæ cum nihil aliud sit quam quadrat. ex $\sqrt{a^2}$ [hoc est $a$] & $\sqrt{a}$, seu $\sqrt{a^3}$; idem est ac $a\sqrt{a}$; ideo primum quæratur $\sqrt{a}$ per præced. XXI. quæ in $a$ ducta planum constituet; quo per XIII. ad rationem lineæ redacto habebitur quæsitum.

Vel data quantitas $a^3$ primum exponatur in lineam, ac tunc fiat extractio juxta XXI.

Notandum autem, pro hujusmodi extractione (uti & in reliquis Casibus hujusce Propositionis) unitatem non posse assumi pro lubitu, sed ab ipsa quantitate $a$ dependere.

2. Sit quærenda $\sqrt{ab^2}$, quæ est = quadrato ex $b$ & $\sqrt{a}$, quo in lineam exposito habebitur quæsitum.

3. Sit quærenda $\sqrt{bcd}$, quæ est = quadrato ex $\sqrt{b}$ & $\sqrt{cd}$. Itaque per XVI. quæritur $\sqrt{cd}$, & per XXI. $\sqrt{b}$; indeque constitutum quadratum exponatur in lineam per XIII. quæ ipsa erit $\sqrt{}$ quæsita.

§. Pro declaratione per numeros esto $a=4$; $b=3$; $c=2$; $d=5$.

## PROP. XXIII.

*Ex data quantitate quatuor dimensionum radicem quadratam extrahere.*

*Nota:* HOC loco idem unitas determinatur per datas quantitates, nec pro lubitu assumi potest.

1. Sit quærenda $\sqrt{a^4}$, quæ est $=a^2$; ideoque $a^2$ per XII. exponatur in lineam & habebitur quæsitum.

2. Sit quærenda $\sqrt{a^2b^2}$, quæ est $=ab$, ideoque ut prius.

3. Sit

3. Sit quærenda $\sqrt{b^2dc}$, quæ est $= b\sqrt{dc}$, quæratur $\sqrt{dc}$ per XVI. quæ cum $b$ constituet planum, quod expositum in lineam per XIII. dabit radicem quæsitam.

4. Sit quærenda $\sqrt{abcd}$; quæratur $\sqrt{ab}$, itemque $\sqrt{cd}$ per XVI. & constitutum ab utraque planum redigatur in lineam per XIII. habebiturque radix quæsita.

## Prop. XXIV.

*Ex data quantitate quatuor dimensionum radicem biquadraticam extrahere.*

1. *Modus qui fit ope unitatis.*

QUÆ hoc loco pro lubitu major minorve assumi potest, semper enim ultimo vera radix prodibit; ac consequenter etiam litera aliqua ipsius datæ quantitatis pro unitate accipi poterit, quod operationi non parum compendii adferet.

*Regula generalis pro omnibus Casibus.*

Quæratur primum per præced. XXIII. $\sqrt{}$ quadr. propositæ quantitatis; ac tum ex ista radice iterum radix extrahatur per XXI. quæ erit $\sqrt{\sqrt{}}$ quæsita.

2. *Modus*

2. *Modus absque ope unitatis.*

1. Pro $\sqrt{\sqrt{a^4}}$ nulla opus operatione, eſt enim ea $= a$.

2. Pro $\sqrt{\sqrt{a^2b^2}} = \sqrt{ab}$ media proportionalis inter $a$ & $b$ dabit quæſitum.

3. Pro $\sqrt{\sqrt{b^2dc}} = b\sqrt{dc}$ media proportionalis inter $b$ & $\sqrt{dc}$ dabit quæſitum.

4. Pro $\sqrt{\sqrt{abcd}} = \sqrt{\sqrt{\overline{ab\sqrt{cd}}}}$ media proportionalis inter $\sqrt{ab}$ & $\sqrt{cd}$ dabit quæſitum.

## PROP. XXV.

*Ex data quantitate quatuor dimenſionum compoſita vel diminuta radicem biquadraticam extrahere.*

HIC itidem ut in præced. unitas quæcunque ad libitum aſſumi poteſt; commodiſſime tamen illa litera quantitatis propoſitæ, quæ in ea plurimum occurrit, unitatis vice fungetur.

1. Pro $\sqrt{\sqrt{\overline{b^2a^2+b^4}}}$. Aſſumpto $b$ pro unitate exponatur per eam $a^2$ in lineam [per XIII.] quæ vocetur $f$. Exponatur item $b^2$ in lineam, quæ erit $= b$; unde $bf = b^2a^2$, & $\sqrt{bf} = \sqrt{b^2a^2}$; ſed & $\sqrt{bf} = a$; cui additâ per XVII. præced. radice $b^4 = b^2 = b$ (quum $b$ ſupponatur $= 1$) habebitur primum $\sqrt{b^2a^2+b^4}$. Ex quæ ſi rurſum [per XXI.]

radix

radix extrahatur habebitur $\sqrt{\sqrt{b^2a^2+b^4}}$ quæsita.

Hinc constructionis compendium: Ex $\sqrt{a^2+b^2}$ extracta radix ope $b=1$, babit $\sqrt{\sqrt{}}$ quæsitam.

2. Pro $\sqrt{\sqrt{a^4+b^4}}$. Posita $a=1$.

3. Pro $\sqrt{\sqrt{bcd^2+bdc^2}}$. Posito $d=1$, &c.

### *Regula generalis hujusce Propositionis.*

Exponantur plana propositarum quantitatum in lineas, per XIII. cum quibus deinde porro procedatur juxta XVI, XVII. Pro radice nimirum quadrata; ac denique juxta XXI. pro rad. Qquadr.

*Sequuntur Constructiones.*

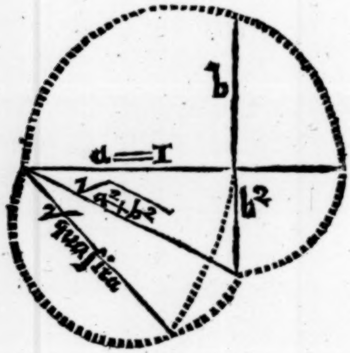
§. Pro $\sqrt{\sqrt{b^2a^2+b^4}}$. Posito $b=1$.


Porro idem obtinebitur quidem si $a$ ponatur $=1$, sed operatione minus compendiosa.

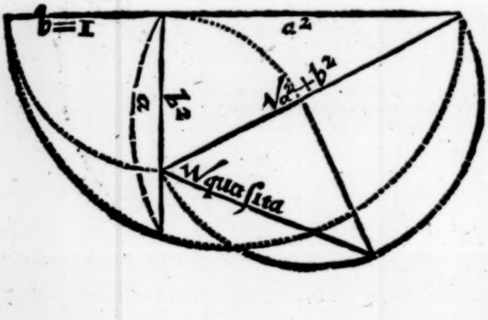
§. Pro

§. Pro $w\sqrt{a^4+b^4}$. 1. Posito $a=1$.

Erit $a^2=a$, jam pro $b^2$, ut $a=1$ ad $b$, ita $b$ ad $\frac{b^2}{1}$ unde acquiritur $\sqrt{a^2+b^2}$; ex qua denuo extracta $\sqrt{\ }$ ope $a=1$, dabit $w\sqrt{\ }$ quæsitam.




2. Posito $b=1$, unde $b^2=b$, &c.



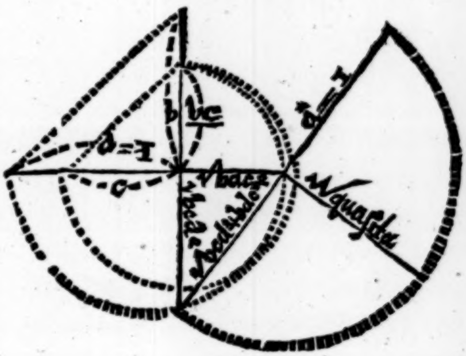

§. Pro

9. Pro $\sqrt{bcd^2 + bdc^2}$.   Posito $d = 1$.

1. Pro $bc$, ut $d = 1$ ad $db$, ita $c$ ad $\frac{bc}{1} = bc$.
2. $d^2 = d$.
3. Med. Proport. inter $bc$ & $d^2 = d$, $= \sqrt{bcd^2}$
4. $bc$ ut ante.
5. $dc = c$.
6. Med. proport. inter $bc$ & $c = dc = \sqrt{bdc^2}$.

Unde 7. habetur $\sqrt{bcd^2 + bdc^2}$.

Ac denique 8. Ope $d = 1$, $\sqrt{}$ quæsita. Quod erat propositum.



Appendix

# APPENDIX
## PRO
## *Numeris irrationalibus.*

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### PROP. XXVI.

*Ex dato numero quocunque radicem quadratam geometrice extrahere, juxta datam ſcalam unitatum.*

Caſus 1.

*Si datus numerus ſit compoſitus.*

MEdia proportionalis inter partes componentes (quaſcunque, ſi propoſitus numerus plures diviſores agnoſcat) dabit radicem quæſitam.

EXEMPLA.

1. Pro $\sqrt{21}$. quær. Med. proport. inter 3 & 7.
2. Pro $\sqrt{30}$. quær. med. proport. inter 2 & 15, vel 3 & 10, vel denique 5 & 6.
3. Pro $\sqrt{99}$. quær. med. proport. inter 3 & 33, vel quod melius, inter 9 & 11.

Caſus

## Casus 2.

*Si datus numerus sit primus:*

Dirimatur is in partes duas pluresve, prout res postulaverit, ita ut partes illæ, quantum fieri possit, quadrata rationalia, vel saltem rectangula, constituant, ac tum procedatur juxta 47. I. &c.

### Exempla.

| | | | |
|---|---|---|---|
| Pro $\surd$ 11 dirimantur | 11 in | 4. 4. 3, vel 9 & 2 | |
| 13 | 13 | 9 & 4 | |
| 29 | 29 | 25 & 4 | |
| 37 | 37 | 25 & 12 | |
| 101 | 101 | 100 & 1 vel 81, 16, 4. | |

Porro in plerisque numeris majoribus, sive compositi sint sive primi, utraque operatio locum habere potest, ut patet ex sequentibus exemplis.

1. Pro $\surd 700$. Subtr. 700 ex 900, restant 200; quorum radice ex supra Cas. 1. quæsitâ, & in semicirculo ex $\surd 900 = 30$ subtractâ, habebitur quæsitum.

2. Pro $\surd 6000$. Med. proport. inter 60 & 100 dabit $\surd$ quæsitam: Vel fiat distributio ut ad figuram constructionis subsequentem videre est.

3. Pro $\surd 4567$. Distributio

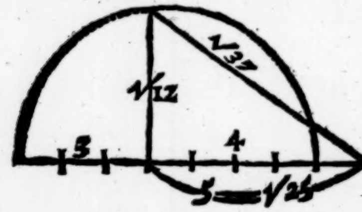
$$\begin{array}{rl} 1936 & \surd\ 44 \\ 2401 & \surd\ 49 \\ \hline 4337 & \text{subtr.} \\ 4567 & \\ \hline 230 & 23 \\ & 10 \end{array}$$

Sequuntur

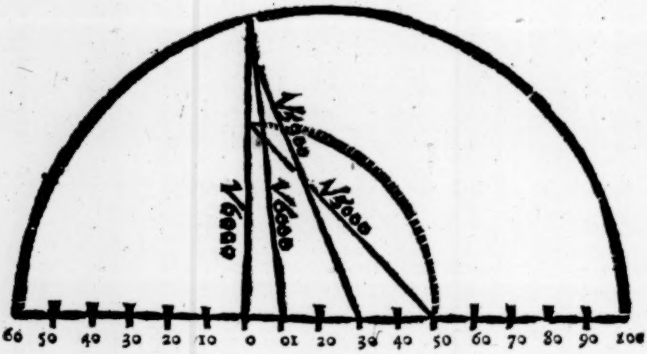
## *Sequuntur Constructiones.*

Pro $\sqrt{21}$.


Pro $\sqrt{37}$.




Pro $\sqrt{6000 \mid 100.}$
$\phantom{Pro \sqrt{}}6\ \ 0$




Vel ita:

6000
50
50
2500
2
5000
30
30
900
10
10
100

add.

PROP:

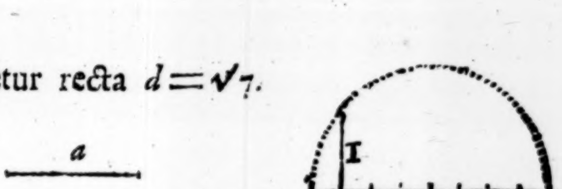
# Prop. XXVII.

*Data recta $=$ radici cujuscunque numeri dati, invenire rectam $=$ unitati competenti seu numero alieni rationali.*

SEcetur recta data in tot partes æquales, quot numerus datus unitates complectitur: Et jam quæratur media proport. inter totam lineam & ejus partem unam; vel inter dimidiam & partes duas, &c. quæ erit ipsa unitas quæsita.

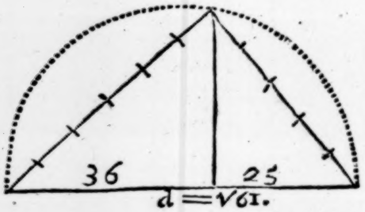
Vel, si numerus datus sit magnus, secetur linea juxta eundem numerum in ea ratione, ut altera pars sit $=$ numero alicui quadrato: Jam inter totam lineam & partem quadraticam dictam quæratur med. proport. quæ erit $\surd$ ipsius quadratici numeri antea designati: Hâc itaque mediâ in tot partes æquales sectâ, quot radix dicti numeri innuit, obtinebitur quæsitum.

1. Detur recta $d = \surd 7$.




2. Detur

2: Detur recta $d = \sqrt{61}$




Secetur $d$ in ratione ut 36 ad 25, quorum quidem uterque quadratus est; ac tum quæratur med. proport. inter totam & partem majorem, prodibit $\sqrt{36} = 6$; vel inter totam & partem minorem, proveniet $\sqrt{25} = 5$.

## Prop. XXVIII. [Consectaria præced.]

*Data recta = radici cujuscunque numeri dati, invenire rectam = radici cujuscunque alterius numeri propositi, idque absque ope unitatis.*

Media proportionalis inter lineam totam & inter tot partes ejusdem quot numerus quæsitæ innuit erit ipsa $\sqrt{}$ numeri propositi quæsita.

Oportet autem datam rectam in tot partes æquales secari, quot numerus datus, cujus radicem data recta refert, unitates continet (prout præced. Prop. dictum) Vel potius ea ratione sectio facienda, ut altera pars sit = alteri numero proposito cujus nimirum radix expetitur.

Casus

## Casus 1.

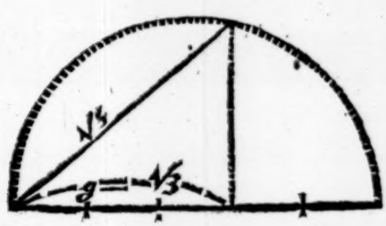
*Ubi linea seu √ quæsita debet esse minor quam data.*

Detur recta $f = \sqrt{5}$; quæratur altera $= \sqrt{3}$.


## Casus 2.

*Ubi linea seu √ quæsita debet esse major quam data.*

Detur vicissim $\sqrt{3} = g$; quæratur $\sqrt{5}$.



# Æquationum Algebraicarum

## *Diſtributio ac Solutio;*

HOC EST,

*De Claſſibus ſeu Ordinibus Æquationum & ſingularum Claſſium Claſſibus ac formulis earumque reſolutione Geometrica.*

ÆQuationum Algebraicarum tot ſunt Claſſes, quot gradus poteſtatum ſeu dimenſionum in ſcala quantitatum progreſſionali. Et tot dimenſionum dicitur aliqua æquatio, quot in quantitate incognita plurimæ reperiuntur; cæteris tamen paribus, hoc eſt, ſi & ſubſequentes gradus quantitatis incognitæ requiſiti adſint, adeoque ipſa æquatio terminis ſuis completa exiſtat: Secus enim per ſuccedentium terminorum, vel termini, abſentiam, æquationes ſæpe ad ſimpliciorem ſeu inferiorem Claſſem deprimuntur, &c.

CLAS-

## Classis I.

### *Æquationum ſimplicium ſeu unius Dimenſionis.*

HArum duo ſunt Caſus:

1. Ubi x æquatur plano diviſo per lineam, vel quicquid huic æquipolleat.
2. Ubi x æquatur radici alicujus plani; vel etiam $\sqrt{\sqrt{\;}}$ alicujus planoplani.

*Prioris Caſus:* [Qui ſpectat quantitates abſolutas ſeu rationales] formulæ ſimpliciſſimæ ſunt

1. $x = \frac{a^2}{b}$  2. $x = \frac{ab}{c}$

Quarum proinde ſolutio expeditur per tertiæ vel quartæ proportionalis inventionem, dicendo nimir.

1. Ut $b$ ad $a$: ita $a$ ad tertiam $= x$.
2. Ut $c$ ad $a$: ita $b$ ad quartam $= x$.

*Poſterioris Caſus*: [Qui concernit quantitates irrationales ſeu radicales] formulæ ſimpliciſſimæ ſunt

| | | | |
|---|---|---|---|
| 1. | $x^2 = d^2$ | Item | $x^4 = d^4$ |
| Unde | $x = d$ | | $x = d$ |
| 2. | $x^2 = bd$ | | $x^4 = b^2d^2$ |
| Unde | $x = \sqrt{bd}$ | | $x = \sqrt{bd}$ |

Harum itaque ſolutio conſiſtit in lateris Quadrati inventione.

Cæterum in solvendis quæstionibus æquationes rarissimæ quidem in ejusmodi simplicissima sui forma emergere solent, sed plerumque ex quantitatibus varie compositis affectisque conflantur: quæ proinde ulterius expendendæ atque ad simplicissimam formulam reducendæ veniunt.

*Sequuntur Exempla.*

Quibus præcipuæ *Æquationum* dictarum (prout emergere solent) *variationes* ac *compositæ formæ* continentur.

Et 1. quidem in *Quantitatibus absolutis* seu *rationalibus.*

1 $bx = 2ac$ Unde $x = \frac{2ac}{b}$

Solutio: ut $b$ ad $2a$: ita $c$ ad x.
Vel: ut $b$ ad $2c$: ita $a$ ad x.

2. $3cx = b^2$ Unde $x = \frac{b^2}{3c}$

3. $2dx = 3a^2$ $x = \frac{3a^2}{2d}$

4. $cx + bx = 2b^2$ $x = \frac{2b^2}{c+b}$

Ut $c+b$ ad $2b$: ita $b$ ad x.

5. $3ax + bx = 6ab$ $x = \frac{6ab}{3a+b}$

6. $dx - ax = 2a^2$ $x = \frac{2a^2}{d-a}$

7. $cx + x = d^2$ $x = \frac{d^2}{c+1}$

8. 2bx

8. $2bx = a^2 + b^2$ Unde $x = \frac{a^2+b^2}{2b}$

9. $ax = ab + b^2$ $x = \frac{ab+b^2}{a}$

10. $2cx = b^2 - a^2$ $x = \frac{b - a^2}{2c}$

11. $3ax = 2bc + dc$ $x = \frac{2bc+dc}{3a}$

12. $dx + cx = ab + bc$ $x = \frac{ab+bc}{d+c}$

13. $2bx - ax = a^2 - \frac{1}{2}ab$ $x = \frac{a^2 - \frac{1}{2}ab}{2b - a}$

14. $3ax - bx = 2b^2 - ac$ $x = \frac{2b^2 - ac}{3a - b}$

15. $2cx = b^2 + c^2 - a^2$ $x = \frac{b^2+c^2-a^2}{2c}$

16. $ax - x = a^2 - b^2 - c^2$ $x = \frac{a^2 - b^2 - c^2}{a - 1}$

17. $ax + cx - \frac{1}{2}dx = a^2 + c^2 - 2d^2$ $x = \frac{a^2+c^2-2d^2}{a+c-\frac{1}{2}d}$

18. $cx = b$ $x = \frac{b}{c}$

19. $bx = 4a$ $x = \frac{4a}{b}$

20. $x = ab$, seu $1x = ab$ $x = \frac{ab}{1}$

21. $2ax = a^2 - b$ $x = \frac{a^2 - b}{2a}$

22. $3x = bc + d$ Unde $x = \frac{bc+d}{3}$

23. $a^2x + b^2x = ac + bc$ $x = \frac{ac+bc}{a^2+b^2}$

24. $dcx - x = 2d^2 + c$ $x = \frac{2d^2+c}{dc-1}$

25. $bdx = c^2a$ $x = \frac{c^2a}{bd}$

26. $c^2x = acd + a^2b$ $x = \frac{acd+a^2b}{c^2}$

27. $2bcx - adx = 3abc + bc^2$ $x = \frac{3abc+bc^2}{2bc-ad}$

28. $bfx + dbx = efg$ $x = \frac{efg}{bf+db}$

29. $a^2x - d^2x = acd - d^2b$ $x = \frac{acd-d^2b}{a^2-d^2}$

30. $b^2dx = a^2c^2$ $x = \frac{a^2c^2}{b^2d}$

31. $2ac^2x - a^2bx = b^4 + 2b^2ac$ $x = \frac{b^4+2b^2ac}{2ac^2-a^2b}$

32. $4b^3x - adx = a^4 - c^4 - c^2a^2$ $x = \frac{a^4-c^4-c^2a^2}{4b^2-ad}$

2. *In Quantitatibus irrationalibus.*

33. $2x^2 = 3ab$ Unde $x = \sqrt{\frac{3ab}{2}}$
seu $x^2 = \frac{3ab}{2}$

34. $x^2 = b^2 + ab$ $x = \sqrt{b^2+ab}$

35. $x^2$

35. $x^2 = b^2 - ab$
36. $4x^2 = 5a^2 - 3bc$
37. $bx^2 = 2a^2c$ $\qquad x = \sqrt{\frac{2a^2c}{b}}$
38. $ax^2 = 3b^2$
39. $bx^2 + x^2 = d^2$
40. $x^2 = a^2b$
41. $x^2 = d^3$
42. $ax^2 - 2bx^2 = ab$
43. $dx^2 = 2ac$
44. $3bx^2 = a^3 + ab^2$
45. $cx^2 + ax^2 = a^2c - b^3$
46. $b^2x^2 = 4a^3$
47. $acx^2 = b^2d^2$
48. $bfx^2 + dbx^2 = e^2gh$
49. $b^2x^2 - d^2x^2 = f^2d^2$
50. $a^2x^2 - \frac{1}{2}bdx^2 = d^4 + 2abd^2$
51. $x^2 = a^2bd$
52. $2ax^2 = d^4$
53. $x^4 = 3bcdf$ $\qquad x = \sqrt[4]{3bcdf}$
44. $dx^4 + x^4 = a^2b^3$
55. $5x^4 = 8ad^3 + c^2ad$
56. $b^2x^4 = b^2c^2f^2 - 2a^6$

## *Ad Æquationes præcedentes Explicatio Casuum quorundam.*

PRimum quod ad usum *Unitatis* attinet, in genere observandum, in omni æquatione requiri, ut Quantitates omnes ab utraque parte numero dimensionum pares existant; alias enim neque rationem neque æqualitatem ad invicem habebunt. Secus itaque si eveniat, hoc est si occurrat Æquatio quæ ab una parte pauciores literas seu dimensiones præferat quam ab altera, aut ubi una quantitatum à communi dimensionum numero deficiat, tunc semper defectum istum adscitâ unitate, eâque in Quantitatem deficientem ducta quoties opus, suppleri intelligendum est: unde si una duntaxat Litera seu Dimensio absit, unitas illa lineam referet; si duæ, Quadratum; si tres Cubum. Hoc itaque pacto Æquationes ejusmodi dimensionum numero coæquandæ sunt, antequam geometrice solvi possint. Et hoc quidem quoties usu venerit, indicio est Problema esse in se Arithmeticum, nec nisi per numeros determinari vel exprimi posse: unde necesse erit, ut datis literis seu lineis, certi numeri, suus cuique, assignentur, quo quidem ipsa simul unitas [ hoc est linea unitatem referens ] determinabitur. Aut si unitas assumatur pro lubitu, tunc semper quantitates datæ quoad valorem numericum ab assumptâ unitate dependebunt, ac consequenter ipsum quæsitum; quod proinde pro alia atque alia unitate variabitur, ut nunc majus nunc minus emergat.

Vel

Vel etiam ita hanc rem exprimere licebit: quum nimirum, uti dictum, ejusmodi Problemata in sua natura sint arithmetica, & vero faciendum, ut geometricè solvi atque exponi possint, necesse erit ut Quantitates numericæ ad formam linearem reducantur; quo quidem fiet ut ipsa quoque unitas sit certa quædam linea: Quæ proinde in alteram lineam numeri alicujus interpretem ducta Quadratum constituet; quod tamen Quadratum non nisi potestate lineari gaudebit. Vid. Prop. V. Logist. Linear.

*Num.* 7. $cx + x = d^2$: hoc est, per observation. præced. $cx + 1x = d^2$: unde $x = \frac{d^2}{c+1}$

Divisor $c+1$ arguit problema esse numericum. Unde vel (1.) Quantitates $c$ & $d$, si sint lineæ, non nisi sub certis numeris dari vel assumi possunt, quo tunc ipsa simul unitas innotescet: Quæ si vocetur $e$, exemplum sic stabit $cx + ex = d^2$, &c. Aut (2.) si unitatem velimus assumere pro lubitu, tunc ipsa illa assumpta unitas lineis $c$ & $d$ suos numeros determinabit, &c.

*Num.* 8. $2bx = a^2 + b^2$. Et $x = \frac{a^2+b^2}{2b}$

Unde (1.) posito $a^2 + b^2$. Seu $x = \frac{a^2}{2b} + \frac{1}{2}b = f^2$ [per XVII. Logist. Lin.] erit.
ut $2b$ ad $f$ : ita $f$ ad $x$:

Vel (2.) ut $2b$ ad $a$: ita $a$ ad $\frac{a^2}{2b}$, adde $\frac{1}{2}b$, habebitur $x$.

*Num.* 9. $ax = ab + b^2$. Et $x = \frac{ab + b^2}{a}$

Seu $x = b + \frac{b^2}{a}$

Quare (1.) poſito $ab + b^2 = g^2$, erit.
Ut $a$ ad $g$ : ita $g$ ad x.

Vel (2.) ut $a$ ad $b$ : ita $b$ ad $\frac{b^2}{a}$; adde $b$, ſumma erit $=$ x.

Vel potius (3.) cum Quantitas $ab + b^2$ ſit orta ex multiplicatione $a + b$ in $b$, —— fiet.

Ut $a$ ad $a + b$ : ita $b$ ad $\frac{ab + b^2}{a} = x$.

*Num.* 10. $2cx = b^2 - a^2$. Et $x = \frac{b^2 - a^2}{2c}$

1.) $b^2 - a^2 = g^2$. Unde, ut $2c$ ad $g$ : ita $g$ ad x.

2.) Ut $2c$ ad $b$ : ita $b$ ad $\frac{b^2}{2c} = d$
Item, Ut $2c$ ad $a$ : ita $a$ ad $\frac{a^2}{2c} = f$
$\Bigg\}$ unde $d - f = x$.

3.) Advertendo Quantitatem $b^2 - a^2$ eſſe compoſitam per multiplicationem $b + a$ in $b - a$; erit ut $2c$ ad $b + a$ : ita $b - a$ ad $\frac{b^2 - a^2}{2c} = x$.

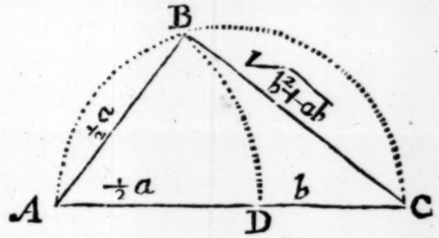
*Num.* 34. $x^2 = b^2 + ab$. Et $x = \sqrt{b^2 + ab}$.

1.) Poſito $ab = d^2$ [per XVI. Logiſt. Lin.] erit $x = \sqrt{b^2 + d^2}$.

Vel

Vel 2.) Observato Quantitatem $b^2+ab$ esse oriundam ex multipl. $b+a$ in $b$, proportio erit, ut $b+a$ ad x : ita x ad $b$. Unde med. proport. inter $b$ & $b+a=x$.

Vel 3.) Construatur triang. rectang. ABC, cujus Hypotenusa AC sit $=\frac{1}{2}a+b$ [nimirum AD$=\frac{1}{2}a$, & DC$=b$] & alterutrum latus circa rectum [ut hic AB] $=\frac{1}{2}a$; tunc enim alterum latus circa rectum, ut BC, erit $=\sqrt{b^2+ab}=x$.

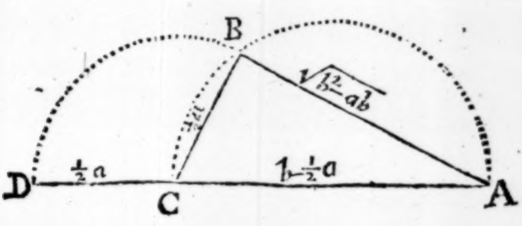



*Num.* 35. $x^2=b^2-ab$. Et $x=\sqrt{b^2-ab}$.

1.) Posito $ab=d^2$, erit $x=\sqrt{b^2-d^2}$.

2.) Ut $b+a$ ad x : ita x ad $b-a$. Unde med. proport. inter $b+a$ & $b-a=x$.

3.) Fiat in triangulo rectang. ABC Hypotenusa AC$=b-\frac{1}{2}a$, & CB$=\frac{1}{2}a$, erit AB$=\sqrt{b^2-ab}=x$.




§. x²

§. $x^2 = b$. Unde $x = \sqrt{b}$.

Hoc loco vel quantitas numero determinanda est, quo innotescat unitas; vel si determinetur unitas, ab ea dependebit quantitas $b$, &c.

N°. 42. $a^2x^2 - 2abx^2 = a^2b^2$. Unde $x = \sqrt{\frac{a^2b^2}{a^2 - 2ab}}$

Vel $ax^2 - 2bx^2 = ab^2$. Unde $x = \sqrt{\frac{ab^2}{a - 2b}}$

Seu ponendo $a - 2b = d$; $x = \sqrt{\frac{ab^2}{d}}$

1. *Modus.* Ponatur $\sqrt{a^2 - 2ab} = f$; erit.

Ut $f$ ad $a$ : ita $b$ ad $\frac{ab}{f} = x$.

2. *Modus.* $x = \sqrt{\frac{ab^2}{d}}$

Ut $d$ ad $b$ : ita $b$ ad $\frac{b^2}{d} = g$. unde $ag = \frac{ab^2}{d}$

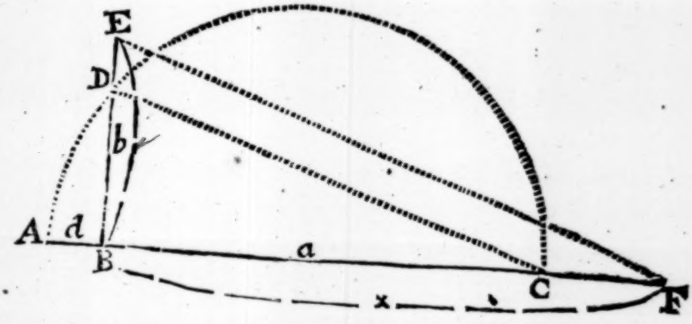
Consequenter $x^2 = ag$. Et $x = \sqrt{ag}$.

3. *Modus.* $dx^2 = ab^2$. Unde proportio.

Ut $d$ ad $a$ : ita $b^2$ ad $x^2$.

Cujus *Constructio*: Fiat [in apposita Figura] $AB = d$ [hoc est $= a - 2b$] & $BC = a$ [hoc est adddantur $d$ & $a$] & describatur super AC semicirculus. Porrò ex B erigatur perpendic. $BE = b$ (hoc est $\sqrt{b^2}$) occurrens peripheriæ in D, tum ducta DC, & huic parallela EF ad prolongatam AC, erit $BF = \sqrt{\frac{ab^2}{d}} = x$.

*Demonstratio.*




*Demnostratio.* Ut AB ad BC [hoc est $d$ ad $a$] ita □BD ad □BC, per *Coroll.* 19. VI. sed ut □BD ad □BC, ita & □BE [$=b^2$] ad □BF [$=x^2$] per 4. & 19. VI. Quare □BE (hoc est $b^2$) ad □BF, ut AB ad BC [hoc est ut $d$ ad $a$] ideoque [per supra expositam proportionem] □BF$=x^2$, & consequenter BF$=$x.

§. Vel brevius ita: BD$=\sqrt{ad}$, & ut BD ad BC: ita BE ad BF. Hoc est ut $\sqrt{ad}$ ad $a$: ita $b$ ad x, unde $x\sqrt{ad}=ab$, & $x^2ad=a^2b^2$, & $x^2d=ab^2$: tum $x=\frac{ab}{\sqrt{ad}}$, sed $\sqrt{ad}=\sqrt{a^2-2ab}$, quoniam $d$ positum $=a-2b$; redit itaque æquatio $x=\frac{ab}{\sqrt{a^2-2ab}}$, quæ erat initiò proposita.

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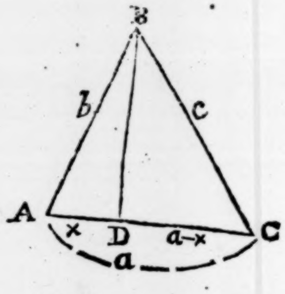
SEQUUNTUR

# SEQUUNTUR

## *Quæstiones* seu *Problemata*

## Nonnulla.

1. DAtis in Triangulo [qualicunque] ABC lateribus, invenire perpendiculum BD [interius cadens]




Sit $AC = a = 14$
$AB = b = 13$
$BC = c = 15$
$AD = x$
$\because DC = a - x$

Jam 1.) In triangulo ABD
$b^2 - x^2 =$ quadrato BD
2.) In triangulo BCD
$c^2 - a^2 + 2ax - x^2 =$ quadrato BD.

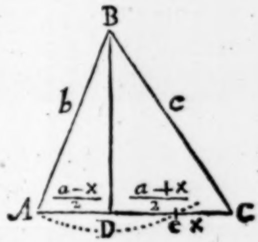
Unde $c^2 - a^2 + 2ax - x^2 = b^2 - x^2$
Et $2ax = b^2 + a^2 - c^2$
Div. $2a$

$$x = \frac{b^2 + a^2 - c^2}{2a}$$

Idem

*Idem aliter.*

Positis cæteris ut supra describatur ex B arcus A$e$.
Fiatque $e$C$=$x ; erit
Igitur DC$=\frac{a+x}{2}$
Et AD$=\frac{a-x}{2}$




Jam 1.) In triangulo ABD.

$\frac{b^2 - a^2 + 2ax - x^2}{4} =$ BD quadrat.

2.) In triangulo BCD.

$\frac{c^2 - a^2 - 2ax - x^2}{4} =$ BD quadr.

Unde $\frac{b^2 - a^2 + 2ax - x^2}{4} = \frac{c^2 - a^2 - 2ax - x2}{4}$

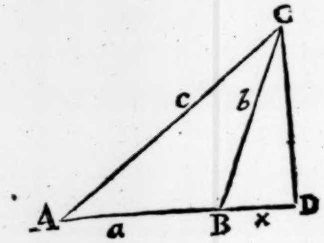
Et $\frac{4ax}{4} = c^2 - b^2$

Hoc est $ax = c^2 - b^2$

$x = \frac{c^2 - b^2}{a}$

2. Datis

2. Datis in triangulo obtuſangulo ABC lateribus, invenire perpendiculum BD [exterius cadens.]




Sit $AB = a = 11$
$BC = b = 13$
$AC = c = 20$
$BD = x$

Jam 1.) In triangulo BCD.
$b^2 - x^2 =$ quadr. CD.
2.) In triangulo ACD.
$c^2 - a^2 - 2ax - x^2 =$ quadr. CD.

Quare $b^2 - x^2 = c^2 - a^2 - 2ax - x^2$
Et $2ax = c^2 - a^2 - b^2$
Div. $2a$

$$x = \frac{c^2 - a^2 - b^2}{2a}$$

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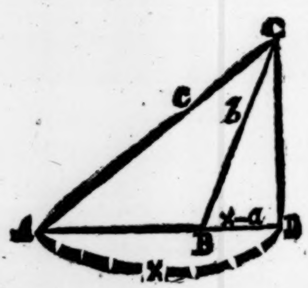
*Idem aliter.*




Poſitis cæteris ut ſupra.

Sit nunc $AD = x$.
Erit $BD = x - a$.

Jam

Jam igitur 1.) In triangulo ACD
$c^2 - x^2 =$ quadr. CD
2.) In triangulo BCD.
$b^2 - x^2 + 2ax - a^2 =$ quadr. CD.

Unde $b^2 - x^2 + 2ax - a^2 = c^2 - x^2$
Et $2ax = c^2 + a^2 - b^2$
Div. $2a$

$$x = \frac{c^2 + a^2 - b^2}{2a}$$

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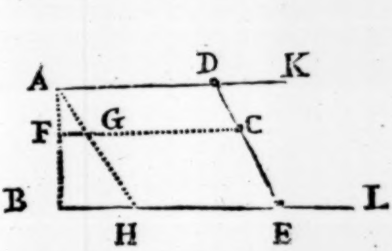
3. Datis duabus rectis AK, BL, parallelis, per datum intra ipsas punctum C rectum ducere DCE, quæ ad data puncta A &B in datis parallelis intercipiat segmenta AD, BE, in ratione data.

Sit AF $= a$
FC $= b$
FB $= c$
AD, GC, HE $=$ x
$\because$ FG $= b - x$




Sitque data ratio $a$ ad $f$
10 18

Quoniam enim dantur duo puncta A & B, licebit rectam AB ducere, & CF parallelam ad AD vel BL; uti & AH ad suppositam DE quæsitam.

 Jam

Jam propter triangula AFG, ABH, ſimilia.

$$\begin{array}{llll} AF & FG & : & AB \quad BH \\ a \div b-x & & : & a+c \mid \dfrac{ab+bc-ax-cx}{a} = BH \end{array}$$

add x=HE, erit $\dfrac{ab+bc-ax-cx+ax}{a}$

Hoc eſt $\dfrac{ab+bc-cx}{a} = BE.$

Inde jam pro obtinenda æquatione.

ut data ratio : ita AD ad BE

$$a \div f \quad : \quad x \div \frac{ab+bc-cx}{a}$$

provenit $fx = ab+bc-cx$
& $fx+cx = ab+bc$
unde $x = \dfrac{ab+bc}{f+c}$

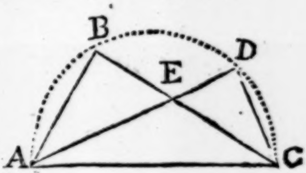
§. Erit itaque proportio æquationis.
ut $f+c$ ad $a+c$ : ita $b$ ad x

Seu rectius hoc ordine:
ut $c+f$ ad $b$ ; ita $c+a$ ad x.

4. Datis

4. Datis duobus triangulis super eadem basi inque eodem circuli segmento, ut ABC, ADC, invenire segmenta mutua laterum, AD, CB.

Sit $AB = a$
$BC = b$
$AD = c$
$DC = d$
$EC = x$
$\because BE = b - x$




Jam propter triangula similia ABE, CDE.

ut AB ad BE : ita CD ad DE

$$a \because b - x : d \mid \frac{bd - dx}{a}$$

Iterum propter eadem

ut CD ad CE : ita AB ad AE

$$d \because x : a \mid \frac{ax}{d}$$

Jam si ex $AD = c$ subtrahatur $AE = \frac{ax}{d}$

remanet $DE = c - \frac{ax}{d}$ hoc est $\frac{cd - ax}{d} = DE$

sed & DE supra inventa $= \frac{bd - dx}{a}$

$$\text{quare } \frac{bd - dx}{a} = \frac{cd - ax}{d}$$

$$\text{\& porro } bdd - ddx = acd - aax$$

$$aax - ddx = acd - bdd$$

$$\text{unde } x = \frac{acd - bdd}{aa - dd}$$

 Jam

Jam [pro constructione] ponendo $aa = df$.

erit $x = \frac{acd - bdd}{df - dd} = \frac{ac - bd}{f - d}$

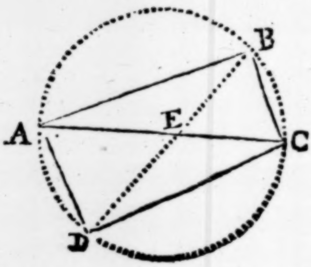
porro ponendo $ac - bd = g^2$, & $f - d = h$

erit $x = \frac{g^2}{h}$

Atque ita proposita æquatio ad formam simplicissimam est reducta.

---

5. Datis duobus triangulis super eadem basi, & in segmentis circuli oppositis, ut ACB, ACD, ductâque ac data diagonio BD, invenire segmenta AE, EC; BE, ED.




Sit $AB = a$
$AC = b$
$BD = c$
$CD = d$
$EC = x$
$\therefore AE = b - x$

Jam

Jam propter triangula ABE, DEC, simil.

1.) Ut DC ad CE : ita AB ad BE

$$d \because x : a \mid \frac{ax}{d}$$

2.) Ut AB ad AE : ita DC ad DE

$$a \because b-x : d \mid \frac{db-dx}{a}$$

Si jam ex BD$=c$ subtr. BE$=\frac{ax}{d}$ erit

$$DE=c-\frac{ax}{d}=\frac{cd-ax}{d}$$

Sed & $DE=\frac{db-dx}{a}$

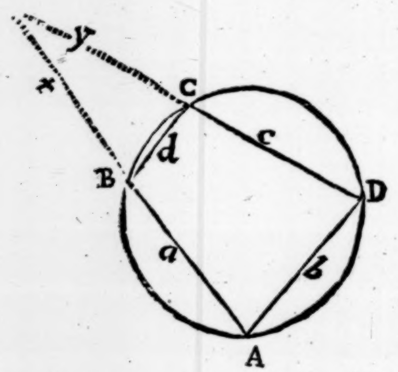
Quare $\frac{db-dx}{a}=\frac{cd-ax}{d}$

Ac denique $x=\frac{acd-bdd}{aa-dd}$

Prorsus ut in præced.

---

6. Datis quatuor lateribus, invenire circulum in quo illa quadrilaterum conſtituant.




Sit $AB = a$
$AD = b$
$CD = c$
$BC = d$
$BE = x$
$CE = y$

Difficultas quippe Problematis in eo conſiſtit ut habeatur BE & CE.

Jam itaque propter triangula AED, CEB ſimil. erit 22. III.

1.) Ut AD ad AE : ita CB ad CE
$b \because a+x : d \because y$

Unde $by = ad+dx$
& $y = \dfrac{ad+dx}{b}$

2.) Ut AD ad DE : ita CB ad BE
$b \because c+y : d \because x$

Unde $cd+dy = bx$
& $y = \dfrac{bx-cd}{d}$

Quare

Quare $\frac{bx - cd}{d} = \frac{ad + dx}{b}$

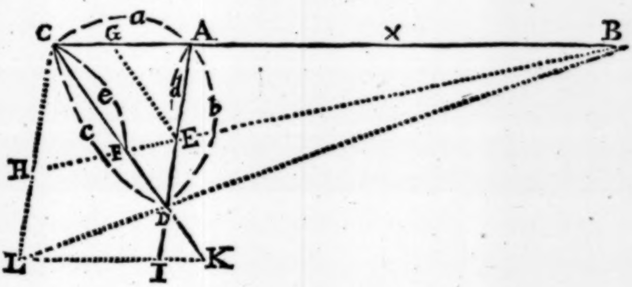
Et porro $b^2x - bcd = ad^2 + d^2x$

$$b^2x - d^2x = ad^2 + bcd$$

$$x = \frac{ad^2 + bcd}{b^2 - d^2}$$

---

7. Invenire distantiam turrium A, B, ex datis AC $= a$, AD $= b$, CD $= c$, AE $= d$, CF $= e$.




Esto AB $= x$

Ducta jam GE parallelâ ipsi CD erit propter triangula ADC, AEG, similia.

1.) Ut AD ad DC : ita AE ad EG.

$b \therefore c : d \mid \frac{cd}{b}$

2.) Ut AD ad AC : ita AE ad AG

$b \therefore a : d \mid \frac{ad}{b}$

add. AB $= x$ erit GB $= \frac{ad + bx}{b}$

 Jam

Jam 3.) Propter ſimil. triangula CBF, GBE.

Ut CF ad CB : ita GE ad GB

$$e \div a+x : \frac{cd}{b} \div \frac{ad+bx}{b}$$

$$\begin{aligned} &\text{Unde} && ade+bex = acd+cdx \\ &\text{\&} && bex-cdx = acd-ade \\ &\text{\&} && x = \frac{acd-ade}{be-cd} \end{aligned}$$

Jam ut æqualitas hæc quam facillime in proportionem reſolvatur, ſimulque eluceat, quo pacto ratiocinandum ſit, pro linea quæſita AB=x, è datis obtinenda, advertere oportet, quænam litera plurimum omnium in hiſce terminis ſeu quantitatibus reperiatur, quæ cum hoc loco ſit $d$, utpote tribus in terminis occurrens, faciendum eſt ut illa etiam in reliquo [$be$] atque adeo in omnibus terminis inveniatur: Quod fiet ſi planum $be$ commutetur in aliud cujus unum latus ſit $=d$, dicendo nimirum

Ut $d$ ad $b$ : ita $e$ ad alterum latus hujus plani, quod latus vocetur $f$ unde $df=be$.

ac conſequenter $\quad x = \dfrac{acd-ade}{df-cd}$

hoc eſt, eliſo ubique $d$: $\quad x = \dfrac{ac-ae}{f-c}$

porro poſito $c-e=g$. & $f-c=h$.

erit $\quad x = \dfrac{ag}{h}$

adeoque $\quad$ ut $h$ ad $a$ : ita $g$ ad x.

Idem

*Idem aliter.*

Potest enim eadem fractio $\frac{acd - ade}{be - cd}$ alia adhuc ratione resolvi: considerando nimirum, in duobus terminis reperiri $cd$, & in reliquis duobus reperiri $e$: itaque si planum $cd$ transmutetur in aliud cujus unum latus sit $e$, litera $e$ sic in omnibus terminis habebitur ac proinde elidetur.

Fiat itaque, ut $e$ ad $c$, ita $d$ ad quartam quæ vocetur $k$, eritque jam $ek = cd$.

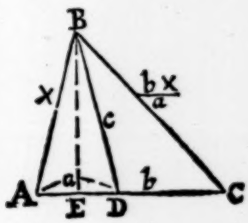
adeoque $\frac{agk - adg}{bg - gk} = \frac{acd - ade}{be - cd}$

hoc est $\frac{ak - ad}{b - k} = x$, &c.

---

8. In triangulo ABC, angulo B secto bifariam per rectam BD, dantur AD, DC, & BD; quæruntur latera AB, BC.

Sit AD $= a$
DC $= b$
BD $= c$
AB $= x$




Jam

Jam 1.) per 3. VI. erit.

Ut AD ad DC : ita AB ad BC

$$a \;::\; b \;:\; \mathrm{x} \;\Big|\; \frac{b\mathrm{x}}{a}$$

2.) per 13. II.

$$\square AD + \square BD - \square AB.$$

$$\frac{a^2 + c^2 - \mathrm{x}^2}{2a} = DE$$

3.) per 12. II.

$$\square BC - \square BD - \square DC.$$

$$\frac{\frac{b^2\mathrm{x}^2}{a^2} - c^2 - b^2}{2b} = DE$$

hoc eſt $\dfrac{b^2\mathrm{x}^2 - a^2c^2 - a^2b^2}{2ba^2} = DE$

unde $\dfrac{b^2\mathrm{x}^2 - a^2c^2 - a^2b^2}{2ba^2} = \dfrac{a^2 + c^2 - \mathrm{x}^2}{2a}$

hoc eſt $\dfrac{b^2\mathrm{x}^2 - a^2c^2 - a^2b^2}{ab} = a^2 + c^2 - \mathrm{x}^2$

& $b^2\mathrm{x}^2 - a^2c^2 - a^2b^2 = a^3b + abc^2 - ab\mathrm{x}^2$

tum $b^2\mathrm{x}^2 + ab\mathrm{x}^2 = a^2b^2 + a^2c^2 + abc^2 + a^3b$

Et dividendo utrobique per $b + a$

$$b\mathrm{x}^2 = a^2b + ac^2$$

unde $\mathrm{x}^2 = \dfrac{a^2b + ac^2}{b}$

& poſito $c^2 = bd$

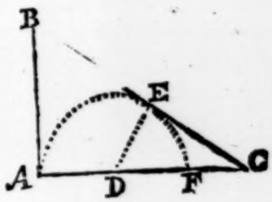
$$\mathrm{x}^2 = a^2 + ad$$

quare $\mathrm{x} = \sqrt{a^2 + ad}$

9. In

9. In triangulo rectangulo ABC descripto semicirculo AEF, tangente latera AB & BC, inveniendum sit latus AB, ex datis AC & DE perpendiculari ad BC ex centro D.

Sit AC $= a$
DE $= b$
AB $= x$
$\because$ FC $= a - 2b$




Jam 1.) Propter triangula simil. BAC, DEC.

Ut AB ad AC : ita DE ad EC.

$$x \;\because\; a : b \;\Big|\; \frac{ab}{x}$$

2.) Per 36. III. □EC=▭ACF.

hoc est, $\dfrac{a^2b^2}{x^2} = a^2 - 2ab$

& $a^2b^2 = a^2x^2 - 2abx^2.$

hoc est, $ab^2 = ax^2 - 2bx^2.$

$$\frac{ab^2}{a-2b} = x^2, \text{ \&c.}$$

Cujus resolutio videatur supra inter formulas æquationum explicatas, *N*°. 42.

10. Sit

10. Sit quærenda linea, cujus Quadratum unà cum rectangulo ſub tota, & ejuſdem parte tertia ſit æquale dato plano $= a^2$.


Eſto linea deſiderata $= x$

$3 = b$

$\therefore \frac{1}{3}$ lineæ $= \frac{x}{b}$

Quare $x^2 + \frac{x^2}{b} = a^2$

$bx^2 + x^2 = ba^2$

Divid. $b + 1$

$x^2 = \frac{ba^2}{b+1}$

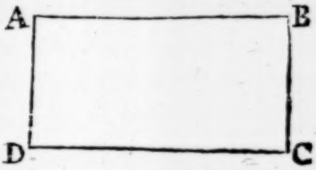
Et ponendo $1 = e$ erit

$bx^2 + ex^2 = ba^2$

$x^2 = \frac{ba^2}{b+e}$

11. Sit

11. Sit rectangulum AC, cujus longitudo AB dupla latitudinis AD, & summa Quadratorum ipsius AB, & AD, decupla summæ eorundem laterum; quæruntur dicta latera AB, AD.




Ut Problema hoc generalius solvatur, sit AD ad AB, ut $e$ ad $f$; & □ AD + □ AB ad AD + AB, ut $g$ ad $e$.

$$\text{Esto jam } AD = x$$

$$\text{erit } AB = \frac{fx}{e}$$

$$\text{\& summa utriusque } = \frac{ex+fx}{e}$$

$$\text{summa autem Quadratorum} = x^2 + \frac{f^2x^2}{e^2}$$

$$\text{seu } \frac{e^2x^2+f^2x^2}{e^2}$$

unde nunc porro per alteram partem hypotheseos erit, ut $g$ ad $e$ : ita $\frac{e^2x^2+f^2x^2}{e^2}$ ad $\frac{ex+fx}{e}$

$$\text{unde } \frac{e^3x^2+ef^2x^2}{e^2} = egx+fgx$$

$$\text{hoc est } e^2x^2+f^2x^2 = egx+fgx$$

$$\text{div. } \overline{\quad x \quad}$$

$$x = \frac{eg+fg}{e^2+f^2}$$

CLASSIS

## Classis II.

### *Æquationum Quadraticarum*, seu *duarum Dimensionum:*

Quo referuntur

*Et* Biquadraticæ *similiter affectæ*; *quippe quarum eadem ratio, nisi quod hic altera radicis extractio superaccedat.*

HUjus Classis Casus ac formulæ simplicissimæ sunt tres:

1.) $x^2 = ax + b^2$

2.) $x^2 = -ax + b^2$

3.) $x^2 = ax - b^2$.

Sic in *Biquadraticis.*

1.) $x^4 = a^2x^2 + b^4$

2.) $x^4 = -a^2x^2 + b^4$

3.) $x^4 = a^2x^2 - b^4$

Quarum singularum *Solutiones* ita se habent.

1.) $x = \sqrt{\tfrac{1}{4}a^2 + b^2} + \tfrac{1}{2}a$

2.) $x = \sqrt{\tfrac{1}{4}a^2 + b^2} - \tfrac{1}{2}a$

3.) $x = \tfrac{1}{2}a \pm \sqrt{\tfrac{1}{4}a^2 - b^2}$

*Ubi not.* Quod tertia hæc formula duas veras radices agnoscat, quarum altera major $x = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}a^2 - b^2}$, altera minor $x = \tfrac{1}{2}a - \sqrt{\tfrac{1}{4}a^2 - b^2}$

Utra

Utra vero in proposita Quæstione aliqua adhibenda sit, ipse usus docebit; quin sæpe utramque suo modo eidem constructioni inservire posse animadvertetur.

Sic in *Biquadraticis.*

1.) $x^2 = \sqrt{\tfrac{1}{4}a^4 + b^4} + \tfrac{1}{2}a^2.$

$x = \sqrt{\sqrt{\tfrac{1}{4}a^4 + b^4} + \tfrac{1}{2}a^2}$

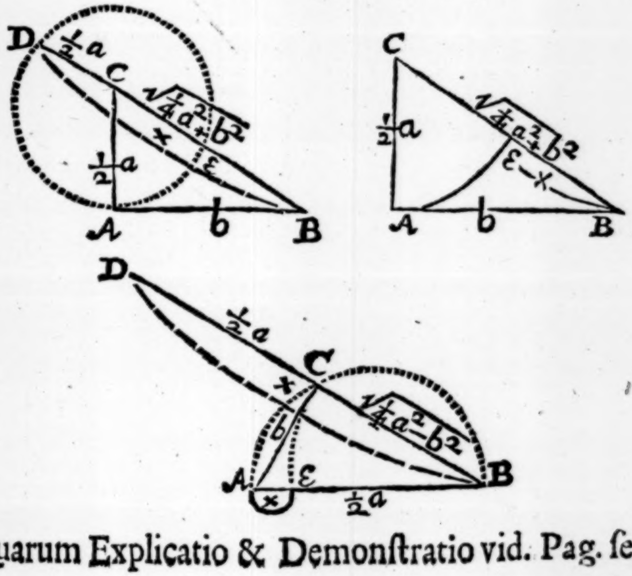
2.) $x^2 = \sqrt{\tfrac{1}{4}a^4 + b^4} - \tfrac{1}{2}a^2$

$x = \sqrt{\sqrt{\tfrac{1}{4}a^4 + b^4} - \tfrac{1}{2}a^2}$

3.) $x^2 = \tfrac{1}{2}a^2 \pm \sqrt{\tfrac{1}{4}a^4 - b^4}$

$x = \sqrt{\tfrac{1}{2}a^2 \pm \sqrt{\tfrac{1}{4}a^4 - b^4}}$

Porro *singulorum Casuum Constructiones* inter cæteras sunt istæ.

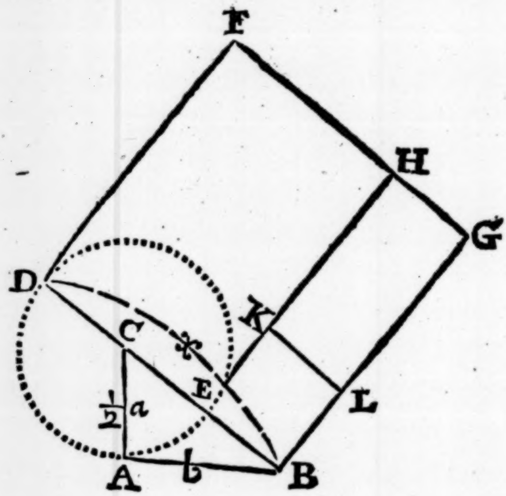



Quarum Explicatio & Demonstratio vid. Pag. seq.

*Demonstra-*

## *Demonſtratio* Primi Caſus.

Sit AC$=\frac{1}{2}a$, & AB$=b$: quibus conjunctis ad angulum rectum in A, deſcribatur centro C, radio CA, Circulus ADE, ductâque recta DCB $=x$, fiat ſuper eadem Quadratum BDFG$=x^2$; agatur porro EH, parallela ipſi BG.




*Dico* 1.) $x^2$ eſſe $=ax+b^2$. Nam primum DCE$=a$, & DF$=x$, componunt rectang. DH$=ax$; deinde per 36.III rectang. DB*e* [hoc eſt GBE] eſt=quadrat. AB, $=b^2$. Quare $ax+b^2=x^2$.

*Dico* 2.) $x$ eſſe$=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$. Nam Quadr. AC eſt$=\frac{1}{4}a^2$, & □ AB$=b^2$: quibus per 47.1 æquatur □BC; ergo BC$=\sqrt{\frac{1}{4}a^2+b^2}$. adde CE=CA $=\frac{1}{2}a$ erit BD = BC + CD. Hoc eſt $x=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$. *q. e. d.*

*Demonſtratio*

## *Demonstratio* Secundi Casus.

Constructâ figurâ ut ante, sit nunc $eB = x$; unde $DB = a + x$ descriptum autem ab $eB$ Quadratum $eL$, $= x^2$.

*Dico nunc* 1.) $x^2$ esse $= b^2 - ax$. Nam $GB = DB = a + x$, & $LB = eB = x$; ergo $GL = De = a$; quæ cum $KL = eB = x$, constituit $\square$ $GK = ax$. Itaque $\square$ $Ge$ [hoc est $\square$ $GK + \square eL$] $= x^2 + ax$. Quod ipsum $\square$ $Ge$, quum Casu præcedenti ostensum sit $= \square AB = b^2$; erit $x^2 + ax = b^2$ & $x^2 = b^2 - ax$.

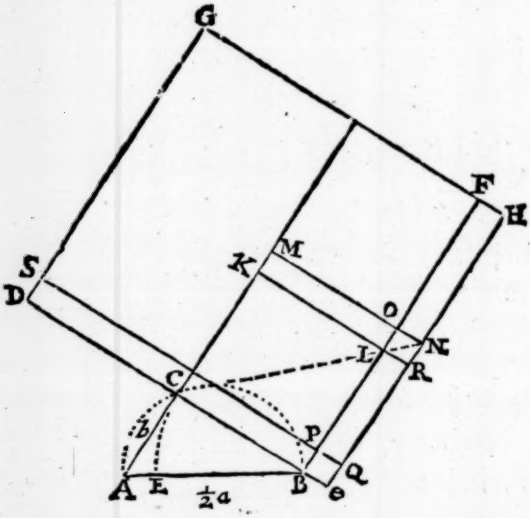
*Dico* 2.) $x$ esse $= \sqrt{\tfrac{1}{4}a^2 + b^2} - \tfrac{1}{2}a$. Nam $Ce = \tfrac{1}{2}a$, & $eB = x$, unde $CB = \tfrac{1}{2}a + x = \sqrt{\tfrac{1}{4}a^2 + b^2}$ per præced. ideóque $x = \sqrt{\tfrac{1}{4}a^2 + b^2} - \tfrac{1}{2}a$. *q. e. d.*

## *Demonstratio* Tertii Casus.

Sit in figurâ sequenti, $AB = \tfrac{1}{2}a$, & $AC = b$. Descripto super AB semicirculo, applicetur AC ad peripheriam, jungaturque BC: quâ prolongatâ fiat CD, $=$ AB.

Sit jam (1.) $DB = x$, tanquam radix major, super qua constitutum $\square$ DF erit $= x^2$. Fiat porro $CE = CD = AB$, agaturque $eH$ parallela ipsi BF, occurrens GF productæ in H, erit $\square$ $DH = ax$.

*Dico* 1.) $x$ esse $= \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2}$ per 47.1 & constructionem.




*Dico* 2.) $x^2$ esse $= ax - b^2$. Constituto enim super $Ce = \frac{1}{2}a$ quadrato CN, erit illud $= \frac{1}{4}a^2$: itemque super $CB = \sqrt{\frac{1}{4}a^2 - b^2}$, descripto quadrato CL, erit hoc $= \frac{1}{4}a^2 - b^2$. Quare gnomon KNB erit $= b^2$: sed gnomoni huic æquatur $=$ ▭ BH, quod sic ostenditur: Ductâ diagonio CN, complementa ML, $e$L, erunt æqualia per 43.1 sed ipsi $e$L est etiam $=$ ▭ OH: Si enim ex BF $=$ BD auferatur BO $= e$N $= e$C, remanebit OF $=$ BC, $=$ BL: Unde cum & OH sit $=$ OK, erit ▭ BH $=$ gnomoni KNB $= b^2$; ac proinde □ DF $=$ ▭ DH $-$ ▭ BH; hoc est, $x^2 = ax - b^2$. *q. e. d.*

Sit

Sit (2.) $Be$, = AE, = $x$, utpote minor: Super qua constitutum quadratum BQ erit = $x^2$, & ▭ $eS = ax$: ex quo si auferatur □BQ, erit reliquum ▭ BS = BH = $b^2$. Unde jam reliqua facile patent.

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Porro *compositæ Æquationum Quadraticarum ac Biquadraticarum formæ* sunt fere hujusmodi; eadem ratione, ut in prima Classe, ad simplices reducendæ.

1. $2x^2 = 4ax - a^2$. Et $x^2 = 2ax - \frac{1}{2}a^2$.

2. $x^2 = 2ax + 2bx - ab$.
ponendo $2a + 2b = f$, erit
$x^2 = fx - ab$

3. $2x^2 = 2ax + b^2 - a^2$.
posito $b^2 - a^2 = d^2$, erit
$x^2 = ax + \frac{1}{2}d^2$

4. $ex^2 = 2adx + 2adf$
$$x^2 = \frac{2adx + 2adf}{e} \text{ \& posito } \frac{2ad}{e} = g$$
$x^2 = gx + gf$.

5. $bx^2 = abx + ac^2$
$$x^2 = ax + \frac{ac^2}{b}$$
posito $\frac{c^2}{b} = d$, erit $\frac{ac^2}{b} = ad$; unde
$x^2 = ax + ad$

 6. $3ax^2$

6. $3ax^2 = b^2x + ab^2$

$$x^2 = \frac{b^2x}{3a} + \frac{b^2}{3} \;\Big|\; \frac{b^2}{3a} = d \text{ ; unde}$$

$$x^2 = dx + \tfrac{1}{3}b^2$$

7. $bx^2 = -d^2x + abf - ad^2$

$$x^2 = -\frac{d^2x + abf - ad^2}{b} \;\Big|\; \frac{d^2}{b} = g \text{ ; unde}$$

$$x^2 = -gx + af - ag \;|\; f - g = k$$

$$x^2 = -gx + ak, \text{ \&c.}$$

8. $5x^2 = 9ax + 9bx + 2b^2 - 4\tfrac{1}{2}a^2$

$$\frac{9a + 9b}{5} = d \;\Big|\; \frac{2b^2 - 4\tfrac{1}{2}a^2}{5} = \pm f^2$$

$$x^2 = dx \pm f^2$$

9. $ax^2 - bx^2 = -a^2x + b^3$

$$a - b = d \;\Big|\; \frac{a^2}{d} = f \;\Big|\; \frac{b^3}{d} = bg \text{ ; unde}$$

$$x^2 = -fx + bg$$

10. $4cx^2 = a^2x + b^2x - 2abx + 4c^2x + 4ab^2$
hoc loco advertendum $a^2 + b^2 - 2ab$ esse Quadratum rationale, cujus radix $a - b$, quæ si ponatur $= d$, erit

$$4cx^2 = d^2x + 4c^2x + 4ab^2, \text{ \&}$$

$$x^2 = \frac{d^2x}{4c} + cx + \frac{ab^2}{c} \quad \text{porro}$$

positis $\frac{d^2}{4c} = f \;\Big|\; f + c = g \;\Big|\; \frac{ab^2}{c} = ak$, erit

$$x^2 = gx + ak.$$

11. $x^2$

11. $x^2 = \frac{rkx + rm^2}{e+f+g+h=n} \Big| \frac{rk}{n} = p \Big| \frac{rm}{n} = q$; unde

$x^2 = px - mq.$

12. $b^2x^2 = - a^2cx + a^2b^2$

$x^2 = - \frac{a^2cx}{b^2} + a^2 \Big| \frac{a^2}{b} = d \Big| \frac{dc}{b} = f$; unde

$x^2 = - fx + a^2$

13. $b^2x^2 = \begin{matrix} 2ac^2x & -a^2c^2 \\ -d^2b & -\frac{1}{4}d^4, \end{matrix}$ &c.

14. $x^2 = \frac{- 2bc^2x + b^2c^2 + a^2d^2 - a^2c^2}{f^2}$

$\frac{2bc^2}{f^2} = g \Big| \frac{b^2c^2}{f^2} = h^2 \Big| \frac{a^2d^2}{f^2} = k^2$

$\frac{a^2c^2}{f^2} = l^2 \Big| h^2 + k^2 - l^2 = m^2$; unde

$x^2 = - gx + m^2$

15. $b^2x^2 - a^2x^2 = 2a^2bx - 2b^3x + a^2b^2$

$a^2 = bd$; unde

$bx^2 - dx^2 = 2a^2x - 2b^2x + a^2b$

$b - d = f \Big| \frac{2a^2 - 2b^2}{f} = g \Big| \frac{a^2}{f} = h$; unde

$x^2 = gx + hh$

16. $a^2x^2 + b^2x^2 = f^2bx + a^2c^2 - \frac{1}{4}f^4$

$a^2 + b^2 = g^2$; unde

$x^2 = \frac{f^2bx + a^2c^2 - \frac{1}{4}f^4}{g^2}$

$$\frac{f^2}{g} = h \;\Big|\; \frac{a^2}{g} = k\text{; unde}$$

$$x^2 = \frac{hhx + kc^2 - \frac{5}{4}hf^2}{g}$$

$$\text{iterum } \frac{hh}{g} = l \;\Big|\; \frac{c^2}{g} = m \;\Big|\; \frac{f^2}{g} = h\text{; unde}$$

$$x^2 = lx + km - \tfrac{1}{2}h^2$$

$$km - \tfrac{1}{2}h^2 = n^2\text{; unde}$$

$$x^2 = lx \pm n^2$$

17. $d^2x^2 - c^2x^2 = -c^2bx + d^2a^2 - c^2a^2 + b^2c^2$

primò divid. $d^2a^2 - c^2a^2$ per $d^2 - c^2$ Quotus erit $a^2$, ponaturque $d^2 - c^2 = f^2$, erit

$$x^2 = \frac{-c^2bx + b^2c^2}{f^2} + a^2$$

$$\text{porro } \frac{c^2}{f} = g\text{, erit}$$

$$x^2 = \frac{-gbx + b^2g}{f}\text{; tandem } \frac{bg}{f} = h\text{, erit}$$

$$x^2 = -hx + hh$$

18. $b^2x^2 - 4x^2 = -8ax + 4a^2 + b^4$, &c.

Subjungi poterant Constructiones formarum Quadraticarum, nisi quæ tum supra *Prop.* XXV. Logist. Lin. dicta sunt, tum sequ. Probl. dicentur, huic rei viderentur sufficere.

*Quæstiones*

# *Quæstiones* seu *Exempla*

## PRO

# ÆQUATIONIBUS

## Quadraticis affectis.

Probl. 1. D*Atum* a *dividere in partes* x *majorem & y minorem, ita ut productum sub partibus sit æquale dato* b.

*Analysis.* Quoniam 1° ex hyp. $x+y=a$ erit $y=a-x$. Et 2° ex hyp. $xy=b$, ergo erit $y=\frac{b}{x}$ ergo cùm $a-x=y$ ($y=$) $\frac{b}{x}$ erit $ax-x^2=b$, sive $x^2=ax-b$. Quare è præmissis erit $x=\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}$.
& $y=\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}$.

Aliter. Quoniam $a-x=y$, multiplicato utrinque per $x$ erit $ax-x^2=xy=$per hyp. ipsi $b$. Quare $ax-x^2=b$, &c.

Aliter adhuc. Erit ex hyp. pars minor $a-x$, quæ si ducatur in majorem $x$ erit productum $ax-x^2=$dato $b$ ex hyp. &c.

*Nota.* Iisdem methodis tam numeri quam magnitudinis investigantur.

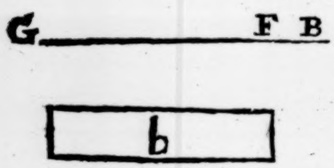
## *Exemplum in Numeris.*

Posito aggregatum $a=20$, & productum $b=36$, & $x=$ majori numero quæsito, erit $20-x=$ minori, & proinde $x$ in $20-x$, sive $20x-x^2=36$, & proinde $x^2=20x-36$. Unde numeri quæsiti ex præmissis eruuntur, scil.

$$x=10+(\sqrt{100-36}=)\ 8=18$$

$$\&\quad y=10-(\sqrt{100-36}=)\ 8=2$$

## *Exemplum in Lineis.*




Detur BG partiendum puta in F, ita ut rectangulum BF+FG sit æquale rectangulo dato $b$.

*Inv.* Sit $BG=a$, majus Segmentum $FG=x$, erit ergo minus Segmentum $BF=a-x$, unde rectangulum sub Segmentis $x$ in $a-x$, sive $xa-x^2=b$. Unde ut prius $x=\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}$:

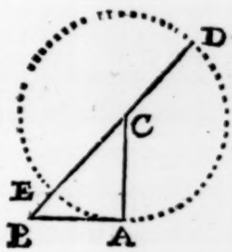
$$\&\quad y=\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}.$$

Quoniam

Quorum omnium *Demonstratio* primo fit intuitu. Nam additis & multiplicatis quæ incognitis $x$ & $y$ inventa sunt æqualia cognita & habebis summam & productum $a$ & $b$, quod erat propositum.

Problema quidem facillimum, tyronum gratiâ, fusius deducitur, tum ut iis minus impeditior foret aditus ad sequentia, tum ut reliquis enucleandis lucem præbeat. Quâ de causâ Constructionem & Demonstrationem ejusdem Geometricam non importunum erit adjungere.

*Constructio.*




Fiat AB medium proportionale inter latera rectanguli $b$; quâ ut basi, & hypotenusâ $BC = \frac{GB}{2}$ fiat triangulum rectangulum ABC; postremo centro C, & radio CA describe circulum AED. Dico BD & BE esse Segmenta quæsita.

*Demonst.* 1° Per Constr. $2BC = GD$. Patet autem $2BC = BD + BE$; ergo inventa BE & BD simul æqualia sunt dato BG.

2° Per Constr. $BA^2 = b$, atqui per (*Eucl.* III. 36) $BA^2 = BE + BD$, ergo rectangulum sub inventis æquatur dato $b$. *Q. E. D.*

Probl.

Probl. 2. *Datis duarum quantitatum producto & differentiâ, easdem invenire singulatim.*

Probl. 3. *Datis duarum quantitatum differentiâ, & quadratorum earundem summâ, quantitates invenire.*

Probl. 4. *Invenire duas quantitates, è datis earum producto, & Cuborum summâ.*

Probl. 5. *E datis, trium continuò proportionalium summâ, & summa quadratorum sub extremis proportionales invenire.*

*Inv.* Sint Prop. $x, y,$ & $z,$ quarum $x$ maxima, proportionalium summa $a,$ & quad. summa $b.$ Quoniam itaque $x, y, z \div$ erit $y^2 = zx$ atqui ex hypothesi $x^2+z^2=b$, quare addito hinc $2y^2$ illinc $2zx$ erit $x^2+2zx+z^2=b+2y^2$, & proinde $x+z=\sqrt{b+2y^2}$. Porrò ex hypoth. $a=y+(z+x=)\sqrt{b+2y^2}$, unde $a-y=\sqrt{b+2y^2}$, & $a^2-2ay+y^2=b+2y^2$, & $y^2=-2ay-b+a^2$. Quare ex præmissis erit $y=\sqrt{2a^2-b}-a$, &c:

Probl.

Probl. 6. *Datis trium continuò proportionalium altero extremo, & summâ quadratorum sub reliquis invenire proportionales.*

Sint $a, x, y \div$ & quadratorum summâ $b$. Erit ex hyp. $ay = x^2$ ergo addito utrinque $y^2$. Erit $y^2 + ay = x^2 + y^2 = b$, ergo $y = \sqrt{\frac{a^2}{4} + b} - \frac{a}{2}$

Probl. 7. *Datis summis, vel differentiis extremorum & mediorum quatuor continuò proportionalium, terminos singulatim investigare.*

Probl. 8. *Dentur, aggregatum quatuor continuò proportionalium, & quadratorum sub terminis mediis vel extremis. Eruendi sunt singuli.*

Probl. 9. *Datam* AB *secare, ut quadratum partis* AE *rectangulo sub totâ & alterâ parte sit æquale.*

*Inventio.* Positis AB $= a$, & AE$= x$, erit pars altera EB$= a - x$; & proinde ex hyp. AE$^2 =$ AB in EB,
sive $x^2 = a$ in $a - x = a^2 - ax$
ideoque $x = \sqrt{+ a\frac{a^2}{4}} - \frac{a}{2}$.

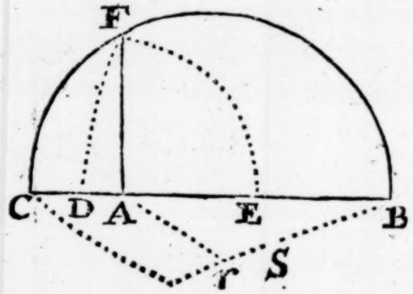

*Constructio.*

*Constructio.* Dimidia AB fiat perpendicularis ad eandem CA, & CD=CB, & AE=AD. Dico factum.

*Demonstratio.* $CA^2+AB^2=CB^2=CD^2=CA^2+(AD)\,EA\times(2CA)\,AB+(AD^2)\,AE^2$, ergo dempto utrinque $CA^2+AE$ in AB erit $AE^2=AB^2-AE$ in $AB=AB\times(AB-AE)\,BE$. Ergo $AE^2=AB$ in EB. *Q.E.D.*

Probl. 10. *Datam* AB *secare ut quadratum partis* AE *ad rectangulum sub totâ & altera parte rationem habeat datam* r *ad* s.

*Inventio.* Sit data $AB=a$
pars quæs. $AE=x$ erit
pars altera $EB=a-x$.
& ex hyp. $AE^2,\ AB\times EB :: r.s.$
i. e. $x^2\ (a \text{ in } a-x)\ a^2-ax :: r.s.$



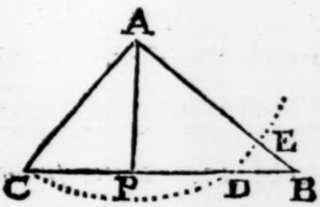

*Constructio.*

*Constructio* 1. Fiat ut in Schemate $s.\ r$ :: AB. AC $= t$. Ergo AC $= \frac{r}{s}$ in AB. Et $x^2 = ta \mp tx$.

Unde $x = \sqrt{ta + \frac{t^2}{4}} \mp \frac{t}{2}$

*Constructio* 2. Descripto semicirculo super BA + AC diametro, ordinatim applicetur AF, & fiat DA = DC & DE = DF. Dico AE esse segmentum quæsitum.

*Demonstratio.* $DA^2 + (AF^2)$ CA in AB $= DF^2 = DE^2 = DA^2 + AE^2 + (2DA)$ CA in AE. Ergo (CA in AB – CA in AE) AC in (AB – AE) EB $= AE^2$; atqui ex *Constr.* 1. AC $= \frac{r}{s}$ in AB; ergo $\frac{r}{s}$ + AB in EB $= AE^2$, & proinde $r.s$ :: $AE^2$ AB in EB. *Q. E. D.*




*Nota.* In sequent. Problemat. rectangulum supponi, cujus hypotenusa notatur literis BC, latus majus BA, minus CA, perpendicularis in hypotenusam AP, segmentum hypotenusæ majus PB, minus CP & eorum differentia DB, differentia autem laterum EB.

*Lemma*

*Lemma* 1. Quadratum hypotenusæ æquari quadratis sub lateribus simul sumptis *Eucl.* I. 48. sive $BC^2 = BA^2 + CA^2$.

*Lemma* 2. Triangula similia, qualia sunt BAC, BPA, APC habere latera æqualibus angulis opposita proportionalia. *Eucl.* VI. *p.* 8. def. 1.

Probl. 11. *Datis differentia laterum* EB, *& perpendiculari in hyp.* AP, *invenire hypotenusam* BC, *&c.*

*Analysis.* Sit $AP = a$, $EB = b$, $CB = x$, & $AC = y$. Per *Lemm.* 1. erit $x^2 = y^2 + (\overline{y+b}^2 =)$ $y^2 + 2yb + b^2 = 2y^2 + 2yb + b^2$. Ergo $x^2 - b^2 = 2y^2 + 2yb$.

Et pr. *Lemm.* 2. BC . CA :: BA . AP.
sive $x \,.\, y :: y+b \,.\, a$

ideoque $ax = y^2 + yb$, ergo $2ax = (2y^2 + 2yb =)$ $x^2 - b^2$. Unde $x^2 = 2ax + b^2$, & è præmissis $a + \sqrt{b^2 + a^2} = x$. *Q. E. I.*

Probl. 12. *Datis summa laterum* AB + AC, *& perpendiculari* AP : *invenire hypotenus.* BC, *&c.*

Probl. 13. *Invenire hyp. & Triangulum è datis differentia segmentorum* DB, *& latere alterutro.*

Probl.

Probl. 14. *Construendum sit Triangulum, è datis differentia segmentorum* DB, *& summa laterum* AB+AC.

Probl. 15. *E differentiâ segmentorum* DB, *& differentiâ laterum* EB *datis, investiganda sunt reliqua.*

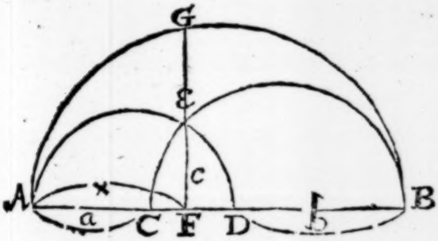
Probl. 16. *E latere alterutro & alterno segmento eruere Triangulum.* Not. *Si detur* AB, *dicetur* PB *segment. alternum.*

Probl. 17. *Datis differentiis inter hypotenusam & latera, sc.* CB—CA *&* CB—BA: *invenire ipsam hypotenusam, &c.*

Probl. 18. *In subjecta figura dantur* AC = a, BD=b, FE=c, *quæruntur diametri singulorum circulorum.*

Sit AF=x

∵ CF=x—a




*Inventio*

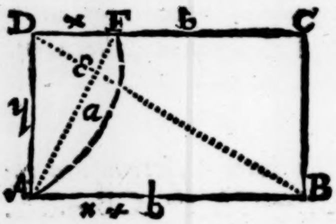
*Inventio* Æquationis facilis est.

provenietque $x^2 = ax + \frac{ac^2}{b}$

ac ponendo $\frac{c^2}{b} = d$, erit

$$x^2 = ax + ad.$$

Probl. 19. *In hoc rectangulo* ABCD, *ductâ* AF *ad angulos rectos per diagonium* BD, *dantur* AF=a *&* FC=b; *quæruntur latera ipsius rectanguli.*




Sit $DF = x$
$DA = y$

Jam 1.) In triangulo ADF

$$y^2 = a^2 - x^2$$

2.) Proport. △ ADF, BAD simil.

$$y \div x : x + b \div y$$

$$\text{unde } y^2 = x^2 + bx$$

$$\text{Quare } x^2 + bx = a^2 - x^2$$

$$2x^2 = -bx + a^2$$

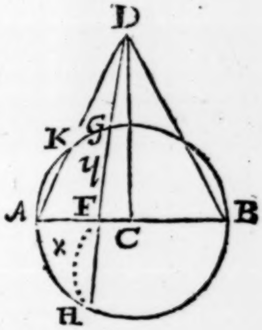
$$x^2 = \frac{-bx + a^2}{2}$$

$$x = \sqrt{\tfrac{1}{16}b^2 + \tfrac{1}{2}a^2} - \tfrac{1}{4}b$$

Probl.

Probl. 20. *Super basi trianguli æquilateri* ABD *descriptus est circulus, ductaque ex vertice* D *recta* DH *per punctum diametri datum* F, *secans peripheriam in* G; *quæritur longitudo ipsius secantis* DH, *ac singulorum ejus segmentorum, puta* DG, GF, FH; *supposito circuli Diametrum* AB *datam esse.*

Sit AC, AK, KD $= a$
CF $= b$
∵ AF $= a - b$
∵ BF $= a + b$
DC $[= \sqrt{3a^2}] = c$
∵ DF $= \sqrt{b^2 + c^2}, = d$
FH $= x$
FG $= y$
∵ DG $= d - y$




Jam 1.) Per 36. III.
$\square$ HDG $=$ $\square$ ADK

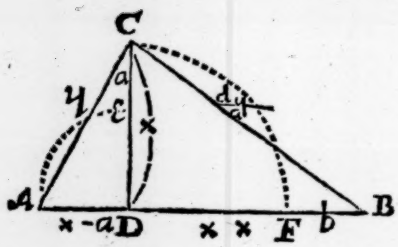
2.) Per 35. III.
$\square$ AFB $=$ $\square$ HFG

Factis faciendis proveniet.

$$x^2 = \frac{3a^2x - b^2x - d^2x + a^2d - b^2d}{d}$$

tum ponendo $3a^2 - b^2 = f^2$
& $f^2 = dg$, erit
$x^2 = gx - dx + a^2 - b^2$, &c.

Probl. 21. *In triangulo* ABC, *demisso ex angulo* C *perpendiculo* CD, *datur differentia utriusque segmenti* AB *&* *perpendiculi* CD, *hoc est*, CE $=$ a, *&* BF$=$b, *una cum ratione lateris* AC *ad* BC *ut* a *ad* d; *quæruntur latera trianguli* ABC.




$$CE = a = 4$$
$$BE = b = 3$$
$$d = 6$$
$$CD = x$$
$$\because\ AD = x - a$$
$$\because\ BD = x + b$$
$$AC = y$$
$$\because\ BC = \frac{dy}{a}$$

Jam 1.) In triangulo ACD

$$y^2 = 2x^2 + a^2 - 2ax$$

2.) In triangulo BCD

$$\frac{d^2y^2}{a^2} = 2x^2 + b^2 + 2bx$$
$$d^2y^2 = 2a^2x^2 + a^2b^2 + 2a^2bx$$
$$y^2 = \frac{2a^2x^2 + 2a^2bx + a^2b^2}{d^2}$$

Unde 3.) $2x^2 + a^2 - 2ax = 2a^2x^2 + 2a^2bx + a^2b^2 \div d^2$

$$2d^2x^2 + a^2d^2 - 2d^2ax = 2a^2x^2 + 2a^2bx + a^2b^2$$
$$\begin{matrix} 2d^2 \\ -2a^2 \end{matrix}\, x^2 = \begin{matrix} 2a^2b \\ +2d^2a \end{matrix}\, x \begin{matrix} +a^2b^2 \\ -a^2d^2 \end{matrix}$$
$$d^2 = af$$
$$\begin{matrix} 2af \\ -2aa \end{matrix}\, x^2 = \begin{matrix} 2a^2b \\ +2d^2a \end{matrix}\, x \begin{matrix} +a^2b^2 \\ -a^2d^2 \end{matrix}$$

Divid.

Divid. per $a$.

$$2fx^2 - 2ax^2 = 2abx + 2d^2x + ab^2$$
$$2f - 2a = g \qquad\qquad - ad^2$$
$$x^2 = \frac{2abx + 2d^2x + ab^2 - ad^2}{g}$$

$$\frac{2ab}{g} = b \quad \Big| \quad \frac{b^2}{g} = l$$
$$\frac{2d^2}{g} = k \quad \Big| \quad \frac{d^2}{g} = m$$

$$x^2 = bx + kx + al - am$$
$$b + k = n \mid l - m = - p$$
$$x^2 = nx - ap$$

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## Constructiones per Numeros.

Pro $af$) $a \div d : d \div f$
$\qquad 4 \quad 6 : 6 \mid 9$

Pro $g$ ) $18 = 2f$
$\qquad 8 = 2a$
$\qquad 10 = g$

$$\S\ x^2 = \frac{2abx + 2d^2x + ab^2 - ad^2}{g}$$

Æquatio hæc pertinet ad Casum 3. quoniam $ad^2$ major quam $ab^2$.

$$\therefore x = \frac{ab + d^2}{g} \pm \sqrt{\frac{\overline{ab + d^2}q}{g^2} + \frac{ab^2 - ad^2}{g}}$$

$$\frac{ab+d^2}{g}=\frac{48}{10}$$

$$\therefore \frac{ab+d^2q}{g^2}=\frac{2304}{100}$$

$$\frac{ad^2-ab^2}{g}=\frac{108}{10}$$

seu $\frac{1080}{100}$

$$\begin{array}{r} 2304 \\ 1080 \\ \hline \frac{1224}{100} \end{array} \text{ subtr.}$$

$$\sqrt{\frac{1224}{100}} = \frac{35}{10} \text{ proximè}$$

$$\frac{48}{10} \text{ add.}$$

$$\frac{83}{10}=x. \; q.e.f.$$

Probl. 22. *Dentur duæ quantitates* $=a$ & $b$; *quærenda sit tertia* $=x$, *ita ut ea addita ipsi* $a$, *summa sit Quadratum, addita vero eadem ipsi* $b$, *summa sit latus dicti quadrati: hoc est* $a+x$ *quadratum*
$\overline{b+x}$ *ejusd. latus*

*Unde* $b^2+x^2+2bx=a+x$.

Jam cum quantitas $a+x$ debeat esse Quadratum, erit ergo superficies per conseq. duarum dimensionum, adeoque in unitatem tanquam lineam ducenda. Assumpta igitur unitate $=b$, erit;

$$b^2+x^2+2bx=ab+bx$$

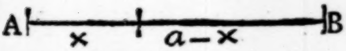
$$\&\quad x^2=-bx+ab-b^2$$

$$\text{unde}\quad x=\sqrt{\tfrac{1}{4}b^2+ab-b^2}-\tfrac{1}{2}b$$

$$\text{hoc est}\quad x=\sqrt{ab-\tfrac{3}{4}b^2}-\tfrac{1}{2}b$$

Probl.

Probl. 23. *Datam rectam* AB, $=a$, *ita secare [extrema quadam & media ratione] ut Quadratum à parte majore, ad rectangulum sub tota & parte minore habeat datam rationem, ut* a *ad* f.




$$a^2+x^2-2ax \qquad ax : a :: f$$

$$fa^2+fx^2-2fax = a^2x$$

$$fx^2 = a^2x+2fax-fa^2$$

$$a^2 = df$$

$$fx^2 = fdx+2fax-fa^2$$

hoc est $x^2 = dx+2ax-a^2$

$$d+2a = 2g$$

$$x^2 = 2gx-a^2$$

Probl. 24. *In triangulo rectangulo* ABC *datur summa laterum circa rectum, hoc est* AB+AC, *cum altitudine* AD; *quæritur triangulum.*

Sit $AB+AC=a$

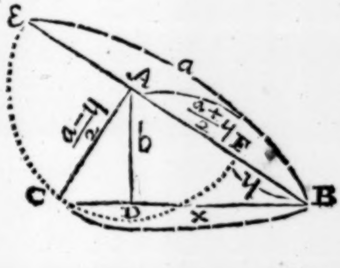
$AD=b$

$BC=x$

$BF=y$

$\because AB=\frac{a+y}{2}$

$\because AC=\frac{a-y}{2}$




 Itaque

Itaque 1.) In triangulo ABC.

$$\square AB + \square AC = \square BC$$

$$\frac{a^2+y^2+2ay}{4} + \frac{a^2+y^2-2ay}{4} = x^2$$

hoc est $\frac{2a^2+2y^2}{4} = x^2$ seu $\frac{a^2+y^2}{2} = x^2$

unde $y^2 = 2x^2 - a^2$

2.) Propter triang. simil. ABC, ADC.

$$\frac{a+y}{2} \div x : b \div \frac{a-y}{2}$$

unde $bx = \frac{a^2-y^2}{4}$

& $y^2 = a^2 - 4bx$

Quare 3.) $2x^2 - a^2 = a^2 - 4bx$

$$2x^2 = -4bx + 2a^2$$

$$x^2 = -2bx + a^2$$

$$x = \sqrt{b^2+a^2} - b$$

*Idem*

*Idem aliter.*

Sit $BC = x$, $CA = y$ | Itaque 1.) In triang. ABC.

$$2y^2 = x^2 - a^2 + 2ay$$

$$y^2 = ay + \frac{x^2 - a^2}{2}$$

$$\text{unde} \quad y = \sqrt{\tfrac{1}{4}a^2 + \tfrac{1}{2}x^2 - \tfrac{1}{2}a^2} + \tfrac{1}{2}a$$

$$\text{hoc est} \quad y = \sqrt{\tfrac{1}{2}x^2 - \tfrac{1}{4}a^2} + \tfrac{1}{2}a$$

2\. Propter triangula simil. ABC, ADC,

$$a - y \div x : b \div y$$

$$\text{unde} \quad bx = ay - y^2$$

$$y^2 = ay - bx$$

$$y = \sqrt{\tfrac{1}{4}a^2 - bx} + \tfrac{1}{2}a$$

[ Notanda hic methodus duarum Æquationum ejusdem formæ ; cujus usus uti pulcherrimus ita nonnunquam omnino necessarius est. ]

$$\text{Quare 3.)} \quad \sqrt{\tfrac{1}{2}x^2 - \tfrac{1}{4}a^2} + \tfrac{1}{2}a = \sqrt{\tfrac{1}{4}a^2 - bx} + \tfrac{1}{2}a$$

& abjecto signo radicali,

$$\tfrac{1}{2}x^2 - \tfrac{1}{4}a^2 = \tfrac{1}{4}a^2 - bx$$

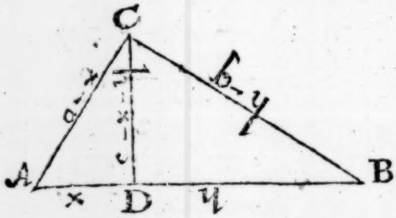
$$\frac{x^2}{2} = -bx + \frac{a^2}{2}$$

$$x^2 = -2bx + a^2 \quad \text{ut supra.}$$

§. Sic etiam si detur differentia dictorum laterum, cum altitudine.

Probl. 25. *In hoc triangulo dantur* $\begin{cases} AC+AD=a \\ BC+BD=b \\ AB+CD=c \end{cases}$

*oportet distinguere singulas.*




Sit $AD=x$

$BD=y$

$\because\ AC=a-x$

$\because\ BC=b-x$

$\because\ DC=c-x-y$

Jam 1.) $\square AC - \square AD = \square BC - \square BD$

$$a^2+x^2-2ax \qquad b^2+y^2-2by$$
$$-x^2 \qquad\qquad -y^2$$
$$a^2-2ax=b^2-2by$$
$$2by=b^2-a^2+2ax$$
$$y=\frac{b^2-a^2+2ax}{2b}$$

2.) $\square AC - \square AD = \square CD$

$$a^2-2ax=c^2+x^2+y^2$$
$$-2cx-2cy+2xy$$

Unde $y^2=2cy-2xy+a^2-c^2-x^2+2cx$
$$-2ax$$

& $y=c-x+\sqrt{a^2-2ax}$

Quare 3.) $c-x+\sqrt{a^2-2ax}=\dfrac{b^2-a^2+2ax}{2b}$

$$2bc-2bx+2b\sqrt{a^2-2ax}=b^2-a^2+2ax$$
$$2b\sqrt{a^2-2ax}=b^2-a^2-2bc+2ax$$
$$+2bx$$

$b^2-$

$$b^2 - a^2 - 2bc = -2d^2$$

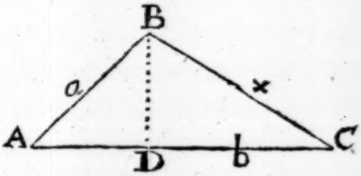
$$2a + 2b = 2f$$

$$2b\sqrt{a^2 - 2ax} = 2fx - 2d^2$$

$$b^2a^2 - 2b^2ax = f^2x^2 + d^4 - 2d^2fx$$

$$f^2x^2 = 2d^2fx - 2b^2ax + b^2a^2 - d^4, \&c.$$

Probl. 26. *In triangulo rectangulo* ABC *dantur* AB $= a$, CD $= b$, *quæruntur cætera.*




1.) $x^2 - b^2 = \square BD$

2.) $a^2 - x^2 + b^2 = \square AD$

3.) $\sqrt{a^2 - x^2 + b^2} + b = \sqrt{a^2 + x^2} = AC$

Unde prodibit

$$x^4 = b^2x^2 + b^2a^2$$

$$x^2 = \sqrt{\tfrac{1}{4}b^4 + b^2a^2} + \tfrac{1}{2}b^2$$

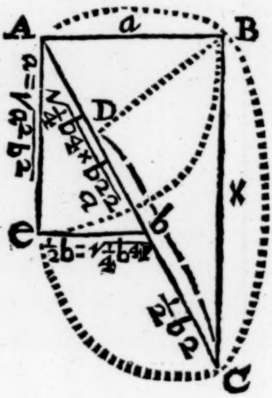
$$x = \sqrt{\sqrt{\tfrac{1}{4}b^4 + b^2a^2} + \tfrac{1}{2}b^2}$$

*Not.* Hæc quæstio faciliori quidem via solvi potest, si nimirum AD ponatur $= x$: At hic pro exemplo æqu. biquadr. esse debuit.

*Itaque*

### *Itaque pro Constructione.*

Assumptâ $b$ pro unitate, erit $b^2 = b$, & $b^4 = b$, & $\sqrt{b^2 a^2} = a$, & $\sqrt{\frac{1}{4} b^4} = \frac{1}{2} b$; unde jam constructio facilis ut sequitur.




Probl.

Probl. 27. *Triangulum* ABC *dividere oportet in duo æqualia, ita ut linea bisectrix* KH *habeat datam longitudinem [possibilem, hoc est, quæ intra maximam & minimam consistat.]*

Vel etiam proponatur ita:

*In dato triangulo datam rectam [idoneam] ita accommodare, ut ipsa triangulum secet in data ratione.*

Sit $AC = a$

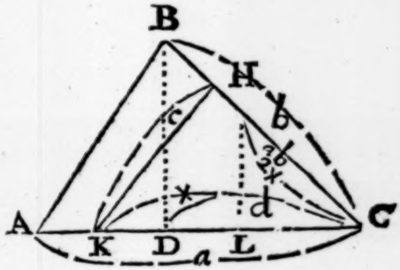
$BC = b$

lin. data $HK = c$

$CD = d$

$CK = x$

$\therefore CH = \frac{ab}{2x}$



Jam 1.) Pro CL per triangula BCD, HCL simil.

$$b \because d : \frac{ab}{2x} \;\Big|\; \frac{abd}{2bx} = CL$$

$$\therefore \frac{2bx^2 - abd}{2bx} = KL$$

2.) $\square HK - \square KL = \square CH - \square CL$

$$\frac{4b^2c^2x^2 - 4b^2x^4 - a^2b^2d^2 + 4ab^2dx^2 = \begin{array}{l} a^2b^4 \\ -a^2b^2d^2 \end{array}}{4b^2x^2}$$

$$4b^2c^2x^2 - 4b^2x^4 + 4ab^2dx^2 = a^2b^4$$

Divid.

Divid. per $4b^2$

$$c^2x^2 - x^4 + adx^2 = \frac{a^2b^2}{4}$$

$$x^4 = c^2x^2 + adx^2 - \tfrac{1}{4}a^2b^2$$

$$c^2 + ad = f^2$$

$$x^4 = f^2x^2 - \tfrac{1}{4}a^2b^2$$

$$x^2 = \tfrac{1}{2}f^2 \pm \sqrt{\tfrac{1}{4}f^4 - \tfrac{1}{4}a^2b^2}$$

$$x = \sqrt{\tfrac{1}{2}f^2 \pm \sqrt{\tfrac{1}{4}f^4 - \tfrac{1}{4}a^2b^2}}$$

Pro *Constructione.*

Quæratur 1.) $\sqrt{ad}$ (2.) $\sqrt{ad + c^2} = f$. (3.) assumpta $f$ pro unitate, erit $\tfrac{1}{2}f = \tfrac{1}{2}f^2 = \sqrt{\tfrac{1}{4}f^4}$. Quær. jam (4.) $\sqrt{\tfrac{1}{2}ab}$, indeque (5.) $\tfrac{1}{2}ab$ [dicendo: ut $f = 1$, ad $\sqrt{\tfrac{1}{2}ab}$ : ita $\sqrt{\tfrac{1}{2}ab}$ ad $\tfrac{1}{2}ab$] $= \sqrt{\tfrac{1}{4}a^2b^2}$. (6.) $\sqrt{\tfrac{1}{4}f^4 \pm \tfrac{1}{4}a^2b^2}$, quæ (7.) add. & subtr. $\tfrac{1}{2}f = \tfrac{1}{2}f^2$. (8.) Ex summa & residuo extracta $\sqrt{}$ ope $f = 1$, dabit radicem æquationis utramque $= x$. Quarum major CM $=$ CK; & minor CN $=$ CH.

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CLASSIS.

## Classis III.

### *Æquationum Cubicarum seu trium Dimensionum.*

Harum Casus numerantur 14: qui tamen sublatione secundi termini reducuntur ad tres vel quatuor, quos numeris suis ad marginem notavimus:

$$x^3 = nx^2 + apx + aaq$$
$$x^3 = nx^2 - apx + aaq$$
$$x^3 = -nx^2 + apx + aaq$$
$$x^3 = -nx^2 - apx + aaq$$
$$x^3 = nx^2 + apx - aaq$$
$$x^3 = nx^2 - apx - aaq$$
$$x^3 = -nx^2 + apx - aaq$$
1. $$x^3 = * + apx + aaq$$
2. $$x^3 = * + apx - aaq$$
3. $$x^3 = * - apx + aaq$$
$$x^3 = nx^2 \; * + aaq$$
$$x^3 = nx^2 \; * - aaq$$
$$x^3 = -nx^2 \; * + aaq$$
4. $$x^3 = * \; * + aaq$$

*Æquatio-*

## *Æquationum Cubicarum & ulteriorum ad solutionem præparatio.*

Quæ sit 1.) Æquationem propositam ad simplicissimam Quantitatum formam reducendo.

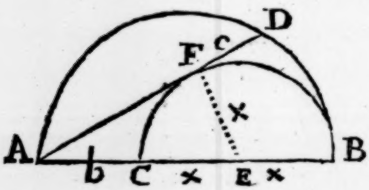
2.) Terminos omnes æquationis reductæ tam cognitos quam incognitos versus unam partem debite coordinando, atque adeo totam Æquationem ponendo $=$ nihilo.

3.) Terminum secundum Æquationis sic coordinatæ tollendo.

4.) Æquationem hac operatione acquisitam correspondenti formulæ applicando, singulosque ejusdem terminos singulis istius coæquando.

*Praxis dictorum exemplaris.*

*Probl.* 25.] In hac figura dantur AC$=b$, FD $=c$; quæruntur cætera.




Posito BE $=$ x

fiat

1.) EB ad FD : ut AE ad AF

$$x \;\cdot\!\cdot\; c \;:\; b+x \;\Big|\; \frac{bc+cx}{x}$$

2.) □AF

2.) $\square AF + \square FE = \square AE.$

$$\frac{b^2c^2+c^2x^2+2bc^2x}{x^2}+x^2=b^2+x^2+2bx$$

$$b^2c^2+c^2x^2+2bc^2x = b^2x^2+2bx^3$$

$$2bx^3 = \begin{array}{l} c^2x^2 \\ -b^2 \end{array} + 2bc^2x + b^2c^2$$

$$x^3 = \frac{\begin{array}{l} c^2x^2 \\ -b^2 \end{array} + 2bc^2x + b^2c^2}{2b}$$

Sequitur 1. *Reductio ad simplicem formam.*

$$x^3 = \frac{\begin{array}{l} -b^2 \\ +c^2 \end{array} x^2 + 2bc^2x + b^2c^2}{2b}$$

ponendo $\frac{-b^2+c^2}{2b} = \mp d$; & $\frac{1}{2}b = f$, erit

$$x^3 = \mp dx^2 + c^2x + fc^2$$

2.) *Coordinatio.*

$$x^3 \pm dx^2 - c^2x - fc^2 = 0$$

3.) *Secundi termini sublatio.*

Ubi *Regula:* Ponatur $x = y$ [ seu loco x assumatur $y$ ] + vel − quantitate cognita secundi termini, divisâ per numerum dimensionum primi: + quidem si secundus terminus habeat −;

ac

*Æquationum Cubicarum & ulteriorum ad solutionem præparatio.*

Quæ fit 1.) Æquationem propositam ad simplicissimam Quantitatum formam reducendo.

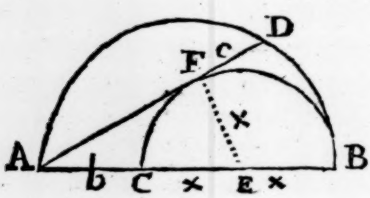
2.) Terminos omnes æquationis reductæ tam cognitos quam incognitos versus unam partem debite coordinando, atque adeo totam Æquationem ponendo = nihilo.

3.) Terminum secundum Æquationis sic coordinatæ tollendo.

4.) Æquationem hac operatione acquisitam correspondenti formulæ applicando, singulosque ejusdem terminos singulis istius coæquando.

*Praxis dictorum exemplaris.*

*Probl.* 25.] In hac figura dantur AC $= b$, FD $= c$; quæruntur cætera.




Posito BE $=$ x

fiat

1.) EB ad FD : ut AE ad AF

$$x \because c : b + x \Big| \frac{bc + cx}{x}$$

2.) □AF

2.) $\square AF + \square FE = \square AE.$

$$\frac{b^2c^2 + c^2x^2 + 2bc^2x}{x^2} + x^2 = b^2 + x^2 + 2bx$$

$$b^2c^2 + c^2x^2 + 2bc^2x = b^2x^2 + 2bx^3$$

$$\begin{array}{l} 2bx^3 = c^2x^2 + 2bc^2x + b^2c^2 \\ \quad\; - b^2 \end{array}$$

$$x^3 = \frac{\begin{array}{l} c^2x^2 + 2bc^2x + b^2c^2 \\ - b^2 \end{array}}{2b}$$

Sequitur 1. *Reductio ad simplicem formam.*

$$x^3 = \frac{\begin{array}{l} -b^2 \\ +c^2 \end{array} x^2 + 2bc^2x + b^2c^2}{2b}$$

ponendo $\frac{-b^2 + c^2}{2b} = \mp d$; & $\frac{1}{2}b = f$, erit

$$x^3 = \mp dx^2 + c^2x + fc^2$$

2.) *Coordinatio.*

$$x^3 \pm dx^2 - c^2x - fc^2 = 0$$

3.) *Secundi termini sublatio.*

Ubi *Regula* : Ponatur $x = y$ [ seu loco x assumatur $y$ ] + vel − quantitate cognita secundi termini, divisâ per numerum dimensionum primi; + quidem si secundus terminus habeat −;

ac viciſſim — ſi is habeat +, adeoque ſub affectione ſignorum contraria; atque tunc fiat quantitatum iſtarum ſubſtitutio conveniens, ut modo ſequetur.

§. Itaque in noſtro exemplo loco x ſumendum erit $y - \frac{1}{3}d$, cum ſecundus terminus ſit affectus ſigno +, & numerus dimenſionum primi termini ternarius.

Sequitur *Operatio.*

$$\left.\begin{array}{l} x = y - \frac{1}{3}d \\ x^2 = y^2 - \frac{2}{3}dy + \frac{1}{9}d^2 \end{array}\right\} \text{mult.}$$

$$\left.\begin{array}{l} y^3 - \frac{2}{3}dy^2 + \frac{1}{9}d^2y \\ \quad - \frac{1}{3}dy^2 + \frac{2}{9}d^2y - \frac{1}{17}d^3 \end{array}\right\} \text{add.}$$

$$\left.\begin{array}{rcrl} x^3 & = & y^3 - & dy^2 + \frac{1}{3}d^2y - \frac{1}{17}d^3 \\ +dx^2 & = & + & dy^2 - \frac{2}{3}d^2y + \frac{1}{9}d^3 \\ -c^2x & = & & - c^2y + \frac{1}{3}dc^2 \\ -fc^2 & & & - fc^2 \end{array}\right\} \text{add.}$$

erit $\quad y^3 * - \frac{1}{3}d^2y - c^2y + \frac{2}{27}d^3 + \frac{1}{3}dc^2 - fc^2 = 0$

ſeu $\quad y^3 = * \begin{array}{l} + \frac{1}{3}d^2 \\ + c^2 y \end{array} \quad \begin{array}{l} - \frac{2}{27}d^3 \\ - \frac{1}{3}dc^2 \\ + fc^2 \end{array}$

Jam cum ultimus terminus conſtet diverſis ſignis ſeu quantitatibus partim negatis partim affirmatis, probe videndum eſt, utrum è dictis ſignis præpolleat, quod hoc loco deprehendetur eſſe ſignum +.

### 4. *Applicatio ad formulam.*

Sublatione ſecundi termini æquationes Cubicæ omnes reducuntur ad ſequentes tres formulas.

$$1.)\ y^3 = * + apy + aaq$$
$$2.)\ y^3 = * + apy - aaq$$
$$3.)\ y^3 = * - apy + aaq$$

Vel ſupponendo $a = 1$.

$$1.)\ y^3 = * + py + q$$
$$2.)\ y^3 = * + py - q$$
$$3.)\ y^3 = * - py + q$$

Cum itaque in acquiſita æquatione noſtri exempli bis recurrat ſignum $+$, facile colligitur eam correſpondere primæ formulæ. Quare jam erit ut ſequitur.

| *Termini ipſius formulæ.* | | *Termini Æquationis propoſ.* |
|---|---|---|
| $y^3$ | $=$ | $y^3$ |
| $+apy$ | $=$ | $+\frac{1}{3}d^2y + c^2y$ |
| & $ap$ | $=$ | $+\frac{1}{3}d^2 + c^2$ |
| unde $p$ | $=$ | $\dfrac{\frac{1}{3}d^2 + c^2}{a}$ |
| $+aaq$ | $=$ | $-\frac{2}{27}d^3 - \frac{1}{3}dc^2 + fc^2$ |
| unde $q$ | $=$ | $\dfrac{+fc^2 - \frac{1}{3}dc^2 - \frac{2}{27}d^3}{aa}$ |

Sequitur *Præparationis* hactenus explicatæ
*Constructio.*

§ Pro $\frac{-b^2+c^2}{2b}$; fiat $\frac{c^2-b}{2b}=-d$, quoniam $b^2$ major.

§ Pro $\frac{\frac{1}{3}d^2+c^2}{a}$; maxime ad rem erit obsequentia, si fiat $\frac{d^2}{a}=g$, & $\frac{c^2}{a}=b$; unde $b+\frac{1}{3}g=\frac{c^2+\frac{1}{3}d_2}{a}=p$.

§ Pro $\frac{d^3}{aa}$ (1.) ut $a$ ad $d$, ita $d$ ad $\frac{d^2}{a}=g$, quæ jam supra est inventa. (2.) Ut $a$ ad $d$, ita $g$ ad quartam $=k$, $=\frac{d^3}{aa}$.

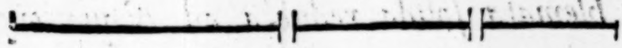
§ Pro $\frac{dc^2}{aa}$ (1.) ut $a$ ad $c$, ita $c$ ad $\frac{c^2}{a}=b$, quæ jam supra est inventa (2.) ut $a$ ad $d$, ita $b$ ad quartam $=l$, $=\frac{dc^2}{aa}$.

§ Pro $\frac{fc^2}{aa}$ [$f=\frac{1}{2}b$] cum hic iterum sit $c^2$, ut supra, erit $\frac{fc^2}{a}=fb$. (2.) Ut $a$ ad $f$, ita $b$ ad quartam $=m=\frac{fc^2}{aa}$.

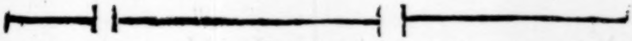
In de jam $m-\frac{2}{27}k-\frac{1}{3}b=q$.

§. Not.

§. *Not.* Ad operationem commodius instituendam assumantur lineæ superioris figuræ magnitudine dupla; sitque *a* = lineæ appositæ.



Peractâ operatione, provenient lineæ ad solutionem necessariæ ut hic videre est.



Fuit hactenus preparatio Æquationum Cubicarum, sequitur dehinc ipsa Solutio.

## *Modus generalis construendi omnia Problemata solida reducta ad Æquationem trium quatuorve dimensionum.*

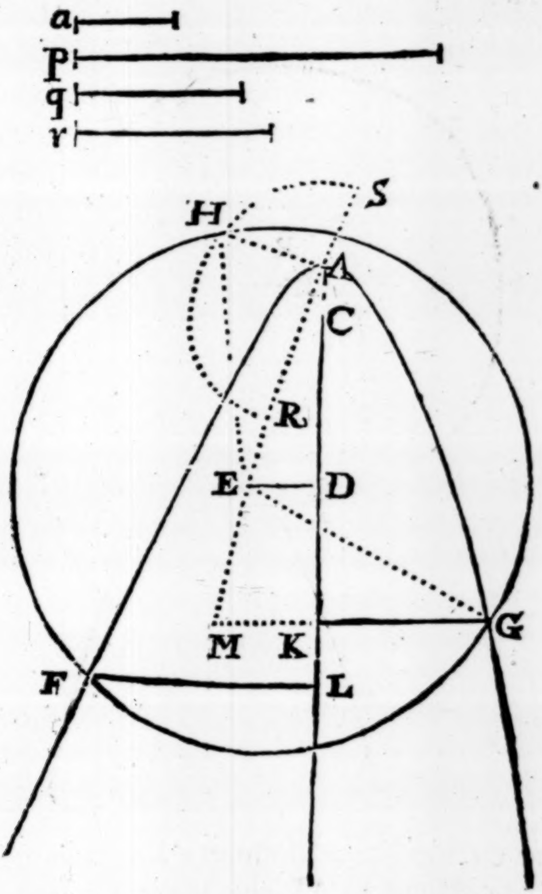
[ Ex Geometr. CARTES, *Lib.* III. *p.* 85. ]

POstquam compertum est Problema propositum esse solidum: sive Æquatio per quam illud quæritur, ad Quadrato-quadratum ascendat; sive non altius quam ad Cubum assurgat: potest semper radix ejus inveniri per aliquam trium Conicarum sectionum, quæcunque illa tandem sit; aut etiam per ipsarum particulam aliquam quantumlibet exiguam, nec adhibendo nisi rectas lineas & circulos. Verum suffecerit regulam generalem hic adducere inveniendi radices omnes ope Parabolæ, quandoquidem hæc aliquo modo est simplicissima.

Primò igitur tollendus est secundus Æquationis propositæ terminus, nisi jam ante absuerit, atque ita Æquatio reducenda est ad hanc formam $z^3 = * \; apz. \; aaq.$ si incognita quantitas tres tantum dimensiones habeat; aut ad hanc $z^4 = * \; apzz. \; aaqz. \; a^3r,$ si quatuor obtineat dimensiones; seu sumendo $a$ pro unitate ad has $z^3 = * \; pz. \; q. \quad z^4 = * \; pzz. \; qz \; r.$

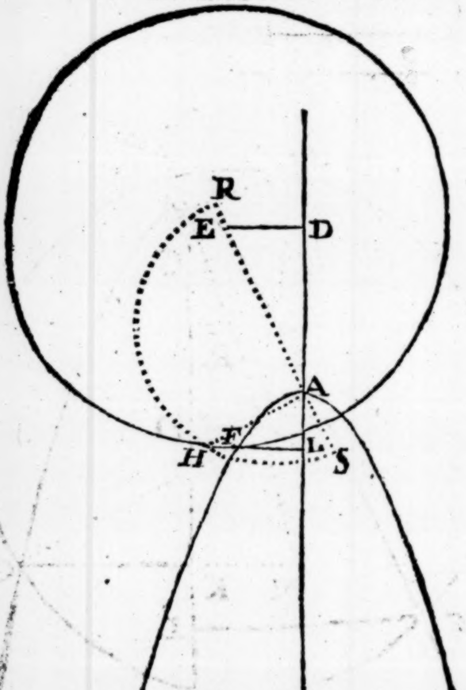
Deinde supponendo parabolam FAG jam descriptam esse, & axem ejus esse ACDKL, latusque rectum $a$ seu 1, cujus AC sit dimidium, & denique

denique punctum C esse intra hanc parabolam cujus vertex sit A. Oportet facere CD $= \frac{1}{2}$ *p*, eamque sumere in linea AC, continuatâ versus C, si in Æquatione habeatur $+$ *p*, sed versus alteram partem si habeatur $-$ *p*. Porrò è puncto D, aut




ex puncto C, si non habeatur quantitas *p*, erigendo ad axem perpendicularem DE $= \frac{1}{2}$ [illegible] oportet

tet ex centro E circulum describere FG, cujus semidiameter sit AE, si Æquatio tantum Cubica fuerit, hoc est, si non habeatur quantitas $r$. Ast si habeatur $r$, & quidem signo $+$ adfecta, oportet ulterius in hac linea AE, producta utrinque, ex una parte sumere $AR = r$, & ex altera parte $AS =$ lateri recto parabolæ quod est $= 1$, descri-

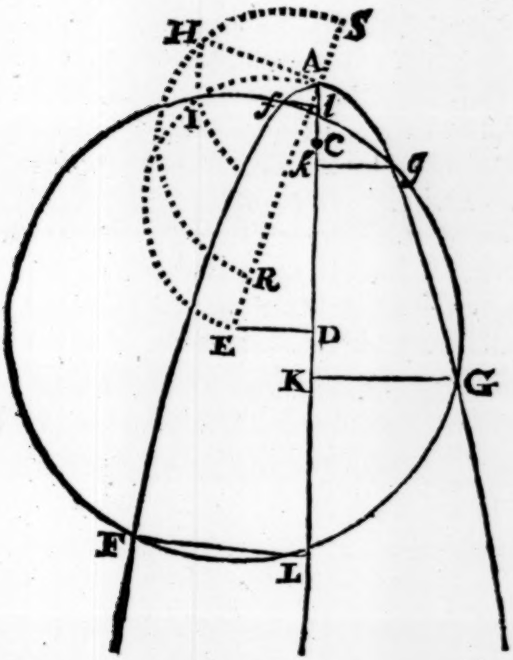



ptoque circulo, cujus diameter RS, erigere AH perpendicularem ad AE, quæ occurrat huic circulo RHS in puncto H; quod illud ipsum est per quod alter circulus FHG transire debet. Quod

si

ſi vero habeatur $-r$, oportet inſuper in alio circulo, cujus diameter eſt AE, inſcribere AI=inventæ AH: inventumque erit punctum I, per quod primus circulus quæſitus FIG tranſire debet.

Ubi ſciendum, quod circulus hic FG ſecare vel tangere poſſit parabolam in 1, 2, 3, aut 4 punctis, à quibus ſi ad axem demittantur perpendiculares, habebuntur omnes Æquationis radices tam veræ



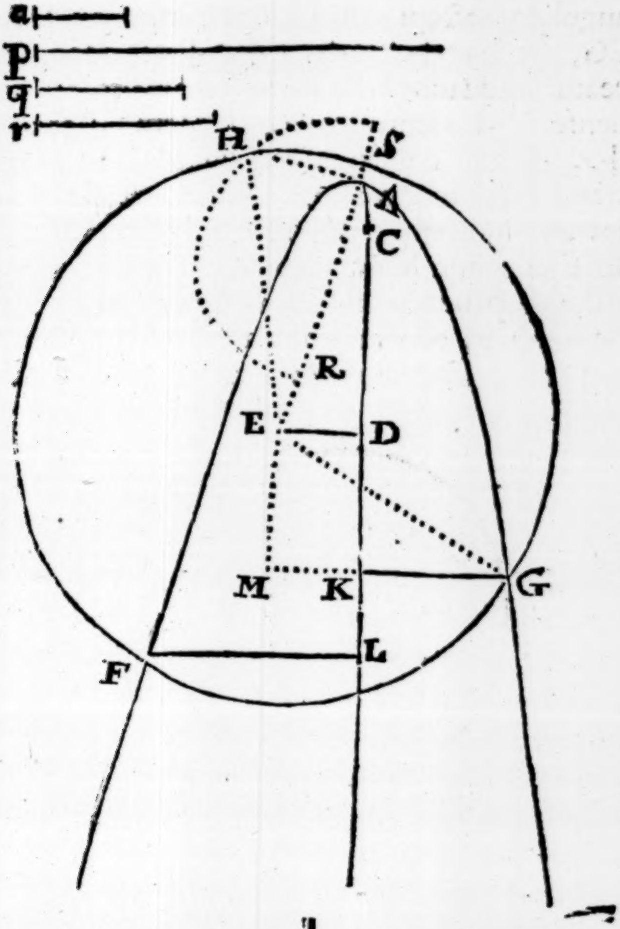

quam falſæ. Nimirum ſi quantitas $q$ ſit adfecta ſigno $+$, veræ radices erunt illæ harum Perpendicularium, quæ ex eadem Parabolæ parte, qua eſt E circuli centrum, reperientur, ut FL; & reliquæ,

liquæ, ut GK, erunt falsæ. Sed contra si hæc quantitas $q$ notata fuerit signo —, veræ erunt illæ, quæ ex altera sunt parte, & falsæ seu minores quàm nihil, quæ ex parte illa ubi est circuli centrum E. Et denique si hic circulus non secat nec tangit Parabolam in aliquo puncto, indicio est Æquationem, nullam admittere radicem sive veram sive falsam sed tantum imaginarias. Adeò ut hæc regula omnium quæ quis exoptare queat, & generalissima sit & perfectissima.

Quorum quidem demonstratio admodum facilis est. Etenim si linea GK, per constructionem hanc inventa, vocetur $z$, AK erit $zz$, propter Parabolam, in qua GK debet esse media proportionalis inter AK & latus rectum, quod est 1. Deinde, si ab AK auferam AC, quæ est $\frac{1}{2}$, ut & CD, quæ est $\frac{1}{2}p$, relinquetur DK seu EM $zz - \frac{1}{2}p - \frac{1}{2}$, cujus quadratum est

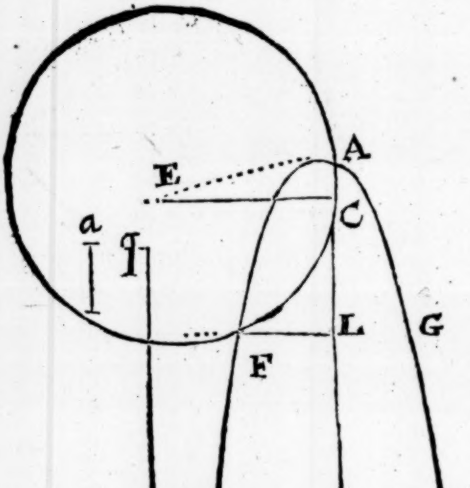
$z^4 - pzz - zz + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4}$. Et quia DE seu KM est $\frac{1}{2}q$, tota GM fit $z + \frac{1}{2}q$, cujus quadratum est $zz + qz + \frac{1}{4}qq$, additísque hisce duobus quadratis, habebitur $z^4 - pzz + qz + \frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4}$, pro quadrato lineæ GE, quippe quæ basis est trianguli rectanguli EMG.

Sed




Sed quia hæc eadem linea GE est semidiameter circuli FG, poterit ipsa aliis adhuc terminis explicari. Nimirum si ED fuerit $\frac{1}{2}q$, & AD $\frac{1}{2}p$, $+\frac{1}{2}$, EA erit $\sqrt{\frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4}}$, propter angulum rectum ADE. Deinde cum AH sit media proportionalis inter AS, quæ est 1, & AR, quæ

quæ est $r$, erit ipsa $\sqrt{r}$. Ac denique, propter angulum rectum EAH, quadratum ex HE seu EG, est $\frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4} + r$: adeò ut habeatur Æquatio inter hanc summam & præcedentem. Eadem quippe quæ $z^4 = * + pzz - qz + r$. Unde consequenter liquet, inventam lineam GK, quæ nominata fuit $z$, Æquationis hujus esse radicem. Quod erat demonstrandum. Et si calculum hunc ad omnes alios hujus regulæ casus applicueritis, mutando signa + & −, prout opus exiget, eodem modo ad quæsitum pervenietis; ita ut illis diutius immorari non sit opus.




Si itaque juxta hanc regulam inter lineas $a$ & $q$ duas libeat medias proportionales invenire, ne-

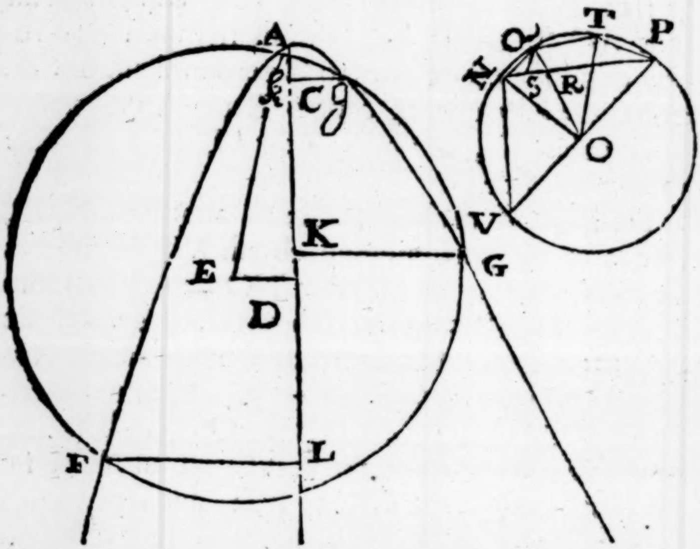
mo ignorat, ponendo $z$ pro una, esse ut $a$ ad $z$, sic $z$ ad $\frac{zz}{a}$, & $\frac{zz}{a}$ ad $\frac{z^3}{aa}$; ita ut habeatur Æquatio inter $q$ & $\frac{z^3}{aa}$, utpote, $z^3 = ** aaq$. Deinde descriptâ Parabolâ FAG, unà cum segmento sui axis AC, quod est $\frac{1}{2} a$, semissis nempe lateris recti, erigenda est ex puncto C perpendicularis CE, æqualis $\frac{1}{2} q$, atque ex centro E per A describendus crculus AF, ut obtineantur FL & LA, duæ mediæ quæsitæ.

Similiter si dividere velimus angulum NOP, sive arcum, portionémve circuli NQTP in tres æquales partes; si sumatur NO $= 1$ pro radio circuli, & NP $= q$ pro subtensa arcus dati, ac NQ $= z$ pro subtensa trientis hujus arcus, exsurget Æquatio $z^3 = * \, 3z - q$. Etenim ductis lineis NQ, OQ, & OT; si QS parallela fiat ipsi TO, patet, quòd, sicut NO est ad NQ, sic NQ sit ad QR, & QR ad RS; adeò ut, cum NO sit 1, & NQ $z$, QR futura sit $zz$, & RS $z^3$. Et quia tantùm RS seu $z^3$ impedit, quò minùs linea NP, quæ est $q$, tripla sit lineæ NQ, quæ est $z$, habebitur

$$q = 3z - z^3, \text{ vel } z^3 = * \, 3z - q.$$

Deinde descriptâ Parabolâ FAG, in qua CA sit æqualis semissi lateris recti principalis, si sumatur CD $= \frac{1}{2}$, & perpendicularis DE $= \frac{1}{2} q$: secabit circulus FA$g$G, centro E per A descriptus, hanc Parabolam in tribus punctis F, $g$, & G, non numerato puncto A, quod est ejus vertex. Id quod indicat in hac Æquatione tres haberi

beri radices, nimirum duas GK & *gk*, quæ veræ sunt, & tertiam, nempe FL, quæ est falsa; atque ex hisce duabus veris minorem *gk* illam esse, quam pro quæsita linea NQ sumere oportet. Altera enim GK, æqualis est ipsi NV, subtensæ




trientis arcus NVP, qui cum reliquo arcu NQP totum circulum complet. Falsa autem FL æqualis est duabus hisce QN & NV simul sumptis, quemadmodum ex calculo facile est videre.

Superfluum foret si insisterem hic aliis exemplis in medium afferendis, cum Problemata omnia, quæ non nisi Solida sunt, eò reduci possint, ut hâc regulâ ad constructionem ipsorum non aliter indigeamus, quàm quatenus inservit ad inveniendas duas medias proportionales, aut ad dividendum angulum in tres æquales partes. Quod cognoscetis, considerando, ipsorum difficultater semper

ſemper Æquationibus, quæ ultra Quadrato-quadratum non adſcendunt, comprehendi poſſe; & omnes illas, quæ ad Quadrato-quadratum adſcendunt, reduci poſſe ad Quadratum, ope quarundam aliarum, quæ tantùm ad Cubum adſcendunt; & tandem, harum ſecundum terminum tolli poſſe. Ita ut nulla earum ſit, quam ad aliquam ex hiſce tribus formis reducere non liceat.

$$z^3 = * - pz + q.$$
$$z^3 = * + pz + p.$$
$$z^3 = * + pz - q.$$

Si autem habeatur $z^3 = * - pz + q$, regula, cujus inventionem Cardanus cuidam, Scipioni Ferreo, tribuit, nos docet, radicem eſſe

$$z = \sqrt{C.\ +\tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p^3}} - \sqrt{C.\ -\tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p^3}}$$

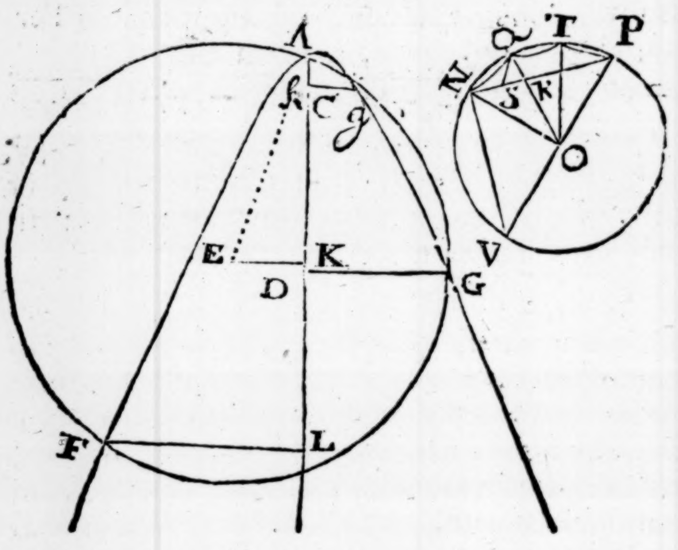
Quemadmodum etiam, ſi habeatur $z^3 = * + pz + q$, & Quadratum ſemiſſis utlimi termini majus ſit Cubo trientis, quantitatis cognitæ penultimi; ſimilis fermè regula nos docet, radicem eſſe

$$z = \sqrt{C.\ +\tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{C.\ +\tfrac{1}{2}q - \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}}$$

Unde apparet, quòd Problemata omnia, quorum difficultates ad Æquationem unius ex hiſce duabus formis reducuntur, conſtrui ſemper poſſint, ut Conicas ſectiones adhibere non ſit opùs, niſi ad extrahendas radices Cubicas ex quibuſdam quantitatibus datis, hoc eſt, ad inveniendas duas medias proportionales inter haſce quantitates & unitatem.

Deinde ſi habeatur $z^3 = * + pz + q$, & Quadratum ſemiſſis ultimi termini non ſit majus Cubo trientis, quantitatis cognitæ penultimi termini; ſupponendo

ſupponendo circulum NQPV, cujus ſemidiameter NO ſit $\sqrt{\tfrac{1}{3}p}$, hoc eſt, media proportionalis inter trientem quantitatis datæ $p$ & unitatem; tum etiam ſupponendo lineam NP huic Circulo eſſe inſcriptam, quæ ſit $\frac{3q}{p}$, hoc eſt, quæ ſit ad alteram quantitatem datam $q$, ut eſt unitas ad trientem ipſius $p$; dividendus tantùm eſt uterque arcus NQP, NVP in tres æquales partes, eritque NQ, ſubtenſa trientis unius arcus, unà cum NV, ſubtensâ trientis alterius, æqualis radici quæſitæ.




Denique ſi habeatur $z^3 = * + pz - q$, ſupponendo rurſus Circulum NQPV, cujus radius NO ſit $\sqrt{\tfrac{1}{3}p}$, & in quo ſcripta NP ſit $\frac{3q}{p}$: erit NQ, ſub-

ſubtendens trientem arcus NQP, una ex radicibus quæſitis: & NV, ſubtendens trientem arcus NVP, radix altera. Saltem ſi Quadratum ſemiſſis ultimi termini non excedat Cubum è triente quantitatis cognitæ penultimi termini. Etenim ſi majus eſſet, non poſſet linea NP huic Circulo inſcribi, quippe quæ diametro ejus major foret. Id quod oſtenderet, duas veras radices hujus Æquationis non niſi imaginarias eſſe, nec ullam realem extare præter falſam, quæ juxta Cardani regulam foret

$$\sqrt{C.\, \tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{C.\, \tfrac{1}{2}q - \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}}.$$

Cæterùm notandum eſt, modum hunc exprimendi valorem radicum per relationem, quam habent ad latera certorum Cuborum, quorum tantùm contentum cognoſcitur, nequaquam magis intelligibilem, neque ſimpliciorem eſſe, quàm ſi exprimantur per relationem, quam habent ad ſubtenſas certorum arcuum, ſeu Circuli portionum, quarum triplum eſt datum. Ita ut Cubicarum Æquationum radices illæ omnes, quæ per Cardani regulas exprimi nequeunt, æquè clarè aut etiam clariùs per modum hìc propoſitum exprimi poſſint.

Si enim, exempli cauſâ, radicem cognoſcere arbitremur hujus Æquationis $z^3 = * + pz + q$: quia ipſam compoſitam eſſe ſcimus ex duabus lineis: quarum una eſt latus Cubi, cujus contentum eſt ſumma, quæ conflatur ex $\frac{1}{2}q$, & ex latere Quadrati, cujus contentum eſt $\frac{1}{4}qq - \frac{1}{27}p^3$; & altera latus alterius Cubi, cujus contentum eſt differentia, quæ eſt inter $\frac{1}{2}q$, & latus Quadrati, cujus contentum eſt $\frac{1}{4}q - \frac{1}{27}p^3$, (quod illud omne

ne est, quod ex Cardani regula addiscimus); Dubitandum non est, quin æquè distinctè aut etiam distinctiùs radix hujus $z^3 = * + pz - q$ cognoscatur, si ea consideretur inscripta Circulo, cujus semidiameter sit $\sqrt{\frac{1}{3}p}$, in quo pro subtensa arcus intelligatur, cujus tripli subtensa sit $\frac{3q}{p}$. Quinetiam hi termini prioribus illis multò minùs sint intricati, & qui etiam multò breviores reddentur, si peculiari aliquâ notâ ad exprimendas hasce subtensas, quemadmodum fit notâ $\sqrt{}$ C. ad exprimendum latus Cubicum, uti velimus.



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FINIS.

Inter Pag. 130 & 131, inserenda.

Idem aliter & facilius, quin etiam generalius.

Producantur BA & ED dum concurrant in M, erit segmentum AD ad segmentum BE, ut MA ad MB: idque semper, puncto C utcunque dato, sive intra sive extra parallelas. Quare si operationis datæ quæratur AM, factum erit quod quærebatur.

Sit igitur nunc,

| | |
|---|---|
| $AB = a$<br>$AM = x$ | & data ratio segmenti minoris AD ad majus BE, ut $f = 10$ ad $a = 18$.<br>Unde MA erit homologa ipsi $f$, & MB homologa ipsi $a$. |

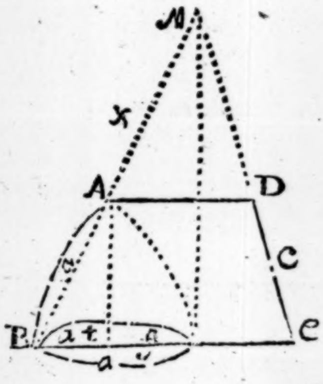
Fiat itaque: Ut data ratio : ita MA ad MB.

$$f \text{ ad } a : \; x \text{ ad } x + a$$

$$\text{erit } ax = fx + af$$

$$ax - fx = af$$

$$x = \frac{af}{a - f}$$




Unde constat constructio.

§ Vel potius,

sit $MB = x$: erit

$$f \because a : x - a \because x$$

$$\text{unde } x = \frac{aa}{a - f}$$

O ERRATA.

# ERRATA sic Corrigas.

PAG. 12. lin. 19 & 20, inter *c* & *d* pone × *vel in*. p. 14, l. ult. $a+3$. p. 15, l. 15, pro $\frac{g}{f}$ lege $\frac{g}{e}$. p. 18, l. 1, 10*a*. p. 20, l. antepenult. *Genesi*. p. 26, l. 11, —*b mult*. l. 13. $2a+b$. p. 55, l. 21, 2*s*. p. 61, l. 17. *Sodales*. p. 64, l. 20. *areolas*. 72, penult. *per* 25. p. 87, l. 5, *grad*. FBC. p. 88, l. 10, *infinitè*. p. 90, l. 5, *grad*. FBC. p. 95. penult. *laterum* [*per unum*]. p. 96, l. 18, *per* V. l. 20, *seu ad valorem linearem*. & l. 27, *assumpto Rectangulo*. p. 101, l. 14, *vel triens*. p. 102, l. 8. =*rectangulo*. p. 104, l. 8, $\sqrt{\overline{\sqrt{ab}\sqrt{cd}}}$. p. 111, juxta Schem. $a=\sqrt{7}$. p. 118, l. 11, $4b^2x$. p. 121, l. 23, *posito* $a^2+b^2=f^2$. *Seu*, &c. $=\frac{f^2}{26}$. p. 123, l. 13 & 14, pro $b+a$ lege $b$. p. 150 l. 3, à fine, *Q. Quadr*. p. 152, l. 14, BF×FG. p. 153, l. 21, 2BC=GB, & penult. =BE×BD. p. 157, l. 12, $\frac{r}{s}\times$AB. p. 172, in Schem. inter D & F dele alteram *x*. p. 177, Not. *hæc methodus*.